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EDITED BY

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THE
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VOLUME XLIV SEPTEMBER 1916 NUMBER 2

THE POLE-EFFECT IN A CALCIUM ARC

By WALTER T. WHITNEY

In the task of establishing accurate standards of wave-length in the spectrum of the iron arc, Goos[1] observed that the wave-lengths of certain lines changed in passing from center to pole of the arc, and that similar changes were caused by increase of current and shortening of the arc. These changes were similar to those caused by increase of pressure, and Goos accounted for the displacements observed on the hypothesis that there existed differences of pressure along an arc and between arcs burning under different conditions.

St. John and Babcock[2] in their work on the pole-effect in the iron arc found displacements of the *d* lines which, on the pressure hypothesis, would demand differences of 1.5 to 2 atmospheres, while certain other lines sensitive to pressure remained unaffected. They state: "It appears, therefore, from the data, that the displacements shown by the lines of groups *c*5, *d*, and *e* are not due to a general increase of pressure in the vapors near the pole of the arc."

Royds[3] suggested that the inconsistencies between Swaim's[4] observations of the pressure-shift of the series lines of zinc and the

[1] *Astrophysical Journal*, **38**, 141, 1913.

[2] *Ibid.*, **42**, 131, 1915.

[3] *Ibid.*, **41**, 154, 1915.

[4] *Ibid.*, **40**, 137, 1914.

general conclusions put forth by Humphreys regarding pressure-shift and series are due in part "to the existence of a density-effect superposed upon the true pressure-effect," since the series lines dealt with are very unsymmetrically widened. The same objection is applicable to the pole-effect measures, but the observations reported in a previous paper,[1] as well as those of St. John and Babcock, point to no change of wave-length with large differences of vapor-density. This point bears more weight in the case of the Ca series lines measured, both first and second subordinate, where much dissymmetry is present.

The purpose of this investigation was to determine the pole-effect for all the principal lines of the Ca spectrum. The photographs were taken in the second order of a 21-foot Rowland concave grating on plates 2×19 inches, which embrace 600 angstroms. The plate scale is 1.32 A per mm. Cramer "Crown" plates were employed in the violet and blue, Cramer "Inst. Iso" in the yellow and green, and Cramer "Spectrum" in the red.

The calcium arc, burning in a horizontal position, was formed between rods of metallic calcium 9 mm in diameter and 2–5 cm long which were firmly clamped in brass holders. In this work the arc was kept about 4 mm long and carried 4 amperes at 110 volts. An image of the arc, enlarged four times, was projected upon the slit of the spectrograph by means of a quartz lens of sufficient aperture to more than completely fill the grating. The negative or cooler electrode was rather sharply pointed and the arc here remained as steady as could be desired. The positive electrode, however, was made with a flat face, since it was found that the arc at the positive pole would not remain at the tip of a pointed terminal but would shift back and forth and finally become extinguished by increasing its length.

In order to avoid overheating of the positive terminal and the consequent rapid oxidation, accompanied by unsteadiness and bad flaring, this electrode was made longer than the negative and surrounded by a small water-jacket (Fig. 1). The positive electrode holder was also mounted in a bearing and provided with a fiber hand-wheel, so that the whole electrode could be rotated through

[1] *Astrophysical Journal*, **43**, 161, 1916.

the water-jacket. In this manner the positive electrode was not only cooled, but as the convection currents carried the arc to the upper side of the positive terminal face the electrode could be rotated to prevent this, and as a result the whole face of the electrode was burned away evenly. This simple device worked a transformation upon a very unsteady arc and since its installation flaring has not been observed. The practical constancy of the arc under these conditions is also well shown by the argreement between the measures from different plates.

Considerable difficulty was at first experienced with mechanical shift, since, on account of the astigmatism of the concave grating, the occulter for producing the comparison spectrum must be

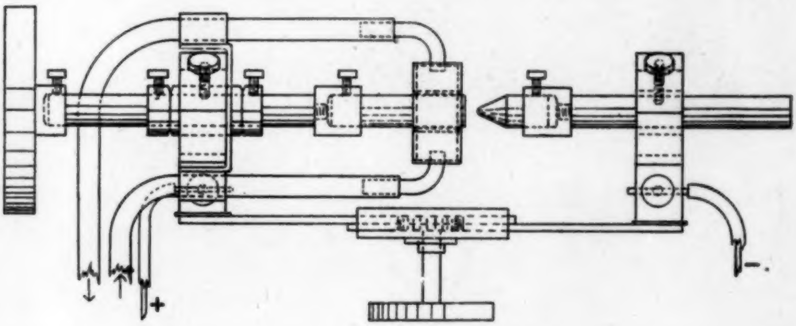


FIG. 1

mounted before the plate and in this case was supported by the camera box which also supported the plate carriage. In this way it was impossible to shift from the central to the comparison spectrum without introducing relatively large displacements of varying sign and magnitude.

A new occulter was therefore constructed (Fig. 2), which consisted of a brass cylinder 53 cm long and 10 cm in diameter. Slots the full length of the photographic plate were cut in the side of the cylinder at *c* and *a*. A wire was drawn through a die to give it the cross-section indicated, and stretched in the middle of the aperture at *c*. This double slot was used for the comparison spectrum. The ends of the cylinder were capped, and the whole was mounted in bearings so as to rotate about a horizontal axis. This

rotation was, however, restricted by the stop-arm which in moving from the stop S_1 to stop S_2 brought the aperture at *a* into the position formerly occupied by the wire in the aperture at *c*. A small slider *d* could be placed between the arm and stop S_2, so that a fourth spectrum might be photographed above the others. This was found to be a great aid in identifying lines due to impurities. A large section was removed from the side of the cylinder toward the grating to permit the light to pass unhindered to the occulter slits and plate. The strips *a* and *b* were made adjustable so that

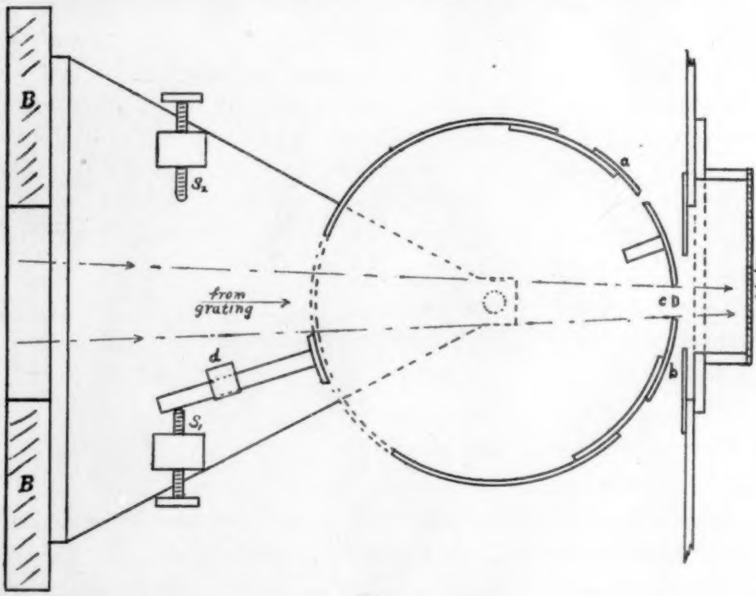


Fig. 2

different widths of spectra could be employed. In this investigation the central and comparison spectra were about 1.5 mm wide and separated by about 0.5 mm. The mounting board *BB* was supported upon clamp stands directly from the cement floor of the grating room, and in this way the whole occulter was mounted independent of the camera, yet much nearer the plate than was the case with the former arrangement.

In order, however, to detect any small instrumental displacements which might occur at times, the following device was

employed. The light from an incandescent bulb was dispersed by means of a small direct-vision spectroscope, and a second slit was placed in the position of the eyepiece. This second slit could be regulated in width, and the color and intensity of the light falling upon it could be chosen at will. An image of it was projected upon the plate, generally near one end, by a good long-focus lens. This furnished a very sharp line which could be accurately measured for plate-shift. All the apparatus involved was mounted directly upon the cement floor and independent of the spectrograph.

It was also found desirable to take all the plates between midnight and morning as at other times the mechanical jars of the building were sufficient to cause considerable plate-shift during exposures. In this manner and with the aid of the new occulter, spectra were obtained which were practically free from plate-shift.

Considerable care was taken to avoid overexposure. This is especially necessary in the case of unsymmetrical lines, as otherwise false photographic shifts of the center of maximum density are easily obtained. In all cases the effort was made, in measuring, to place the cross-hair of the comparator microscope upon the position of maximum density in the photographic image.

The spectra were measured on a small Gaertner comparator provided with a screw of 0.5 mm pitch, the smallest division on the large graduated head reading 0.001 mm. By using the narrow spectra described above, it was possible to increase the magnification as far as the plate grain would permit, and in this way to increase somewhat the accuracy of measurement.

Table I contains the data from the water-cooled arc. The first column gives the wave-lengths and series designation, if any. Saunders'[1] notation has been followed with the exception that in the cases of the two subordinate series and the narrow triplet series a subscript has been added to indicate the number of the group in the series formula. It has not been possible to detect any systematic difference in the pole-effect for the different lines of any single triplet, and these measures have been averaged and the mean taken as the shift for the group. Column two gives the intensity and character of the lines; "h" indicates a hazy line and "hR" or "hV"

[1] *Astrophysical Journal*, **32**, 152, 1910.

TABLE I

POLE-EFFECT FOR CALCIUM. ATMOSPHERIC PRESSURE (75 CM, 20° C.)

λ Series	Int.	Pos. − Center	No.	Av. Diff.	Neg. − Center	No.	Av. Diff.
6499.8 p.....	3	+0.005A	4	0.001A	0.000A	4	0.001A
93.9 p.....	7	+ .003	4	.002	.000	4	.002
71.8.......	4	+ .004	4	.001	.000	4	.001
62.7.......	8	+ .003	4	.001	− .002	4	.001
49.9 p.....	4	+ .003	4	.001	− .003	4	.001
39.3.......	8	.000	4	.001	− .001	4	.001
6169.8.......	3	+ .017	4	.001	.000	4	.001
62.4 T2$_3$..	9						
22.4 T2$_3$...	8	+ .017	9	.001	− .001	9	.001
02.9 T2$_3$...	7						
5857.7.......	8hR	+ .032	4	.003	+ .001	4	.001
5603.1.......	6	+ .011	4	.003	− .003	4	.003
01.5.......	6	+ .011	4	.004	− .003	4	.003
5598.6.......	6	+ .008	4	.001	− .003	4	.001
94.6.......	8	+ .011	4	.004	− .001	4	.003
90.3.......	6	+ .009	4	.003	− .003	4	.003
88.9	8	+ .008	4	.003	− .004	4	.003
82.1.......	9	+ .009	4	.001	− .003	4	.003
13.1	6hV	− .032	4	.001	− .003	4	.001
5349.6.......	8	+ .007	4	.001	− .003	4	.001
5270.4.......	8	+ .018	4	.001	.000	4	.001
65.7.......	8	+ .016	4	.001	.000	4	.001
64.4	6	+ .014	4	.003	.000	4	.001
62.4.......	6	+ .016	4	.003	.000	4	.001
61.9	6	+ .016	4	.003	.000	4	.001
5189.0.......	7	− .007	4	.001	+ .003	4	.001
5041.9 SL2..	7hR	+ .024	6	.004	− .003	6	.001
4878.3 SL3...	8hR	+ .045	4	.004	.000	4	.001
4685.4.......	4hV	− .012	1	....	+ .002	2	.001
4586.1 t$_4$.....	6hR						
81.6 t$_4$.....	5hR	+ .040	6	.005	− .004	9	.001
78.8 t$_4$.....	4hR						
4527.1 SL2..	4hR	+ .036	6	.003	− .008	6	.001
4456.1 T1$_4$...	6						
54.9 T1$_4$...	9						
35.8 T1$_4$...	6	− .003	30	.003	+ .001	30	.003
35.1 T1$_4$...	8						
25.6 T1$_4$...	7						
4355.4*.......	3hR	....	....	....	....	....	....
4318.8 T.....	6hV	− .003	6	.001	+ .001	6	.001
07.9.......	6	− .001	6	.001	.000	6	.003
02.6 p.....	7	− .001	6	.001	.000	6	.003
4299.1 T.....	5hV	− .003	6	.001	.000	6	.001
89.5 T.....	6hV	− .001	6	.003	+ .001	6	.003
83.1 p.....	6	− .003	6	.003	.000	6	.003
4240.5†	3h	....	....	....	....	....	....
4226.9.......	15	+ .005	6	.001	.000	6	.001
4098.6 t$_5$....	5hR						
95.0 t$_5$.....	4hR	+ .055	4	.007	− .013	6	.004
92.7 t$_5$....	3hR						
3973.8 T2$_4$...	7hR						
57.2 T2$_4$...	6hR	+ .024	12	.005	− .003	12	.003
49.1 T2$_4$...	5hR						

* λ 4355 was too diffuse for measurement.

† λ 4240 was masked by λ 4226.

TABLE I—*Continued*

λ Series	Int.	Pos.—Center	No.	Av. Diff.	Neg.—Center	No.	Av. Diff.
H 3968.6 PH.	20	+0.002A	10	0.003A	0.000A	10	0.004A
K 33.8 PH.	25	+ .002	10	.001	.000	10	.003
3737.1 P_2..	8	+ .012	10	.004	+ .001	10	.003
06.1 P_2..	9	+ .012	10	.005	+ .001	10	.004
3644.8 $T1_5$.	5hV						
44.5 $T1_5$.	8hV						
31.1 $T1_5$.	4hV	− .037	15	.009	+ .005	15	.003
30.8 $T1_5$.	7hV						
24.1 $T1_5$.	6hV						
3487.7 $T2_5$.	5hR						
74.9 $T2_5$.	4hR	+ .030	16	.005	− .003	9	.001
68.6 $T2_5$.	3hR						
3361.9 $T1_6$.	7hV						
50.2 $T1_6$.	6hV	− .050	11	.010	+ .011	12	.004
44.4 $T1_6$.	5hV						
3286.2 $T2_6$.	4hR						
74.8 $T2_6$.	3hR	+ .034	4	.005	− .005	4	.001
69.3 $T2_6$.	2hR						
3181.4.....	7	+ .008	6	.003	− .003	4	.003
79.4 P_1..	8	+ .008	8	.004	+ .001	8	.003
58.9 P_1..	9	+ .010	8	.003	.000	6	.003

The lower members of the subordinate series groups were too diffuse for measurement.

one unsymmetrically widened toward the red or violet. Column three contains the measured pole-effect between center and positive. The sign has been chosen such that + indicates a greater wave-length at the pole. Column four gives the number of observations and column five the average difference from the mean. The last three columns contain the data for the negative pole.

It is to be noted from Table I that in general the shift at the negative pole was less than that at the positive pole and of opposite sign. This fact is very striking, in connection with the subordinate series lines where the shifts are relatively large, and is in marked contrast to the observations of Goos and of St. John and Babcock in the iron arc. These observers found that for the lines of the *d* and *e* groups in the spectrum of iron the shift in going from the center of the arc to the negative pole was in the same direction and greater than that found in going from the center of the arc to the positive pole. St. John and Babcock state that a longer exposure was required at the center of the arc than at either pole in order to secure lines of the same strength. In the calcium arc a longer exposure was required at the negative pole than at the center of the

arc, and a longer exposure at the center than at the positive pole. At the positive pole practically all lines are about three times as strong as in the center of the arc. More variation, however, was observed at the negative pole. Here the triplets and single-line series have about one-half the intensity shown in the center of the arc. Many of the non-series lines show no difference in intensity between center and negative pole, while the pair series lines show a greater intensity at the negative than in the center of the arc. All lines are broadened and hazy at the positive pole, and in the case of the pair series lines, although they are stronger at the negative pole than in the center of the arc, they are nevertheless sharper at the pole.

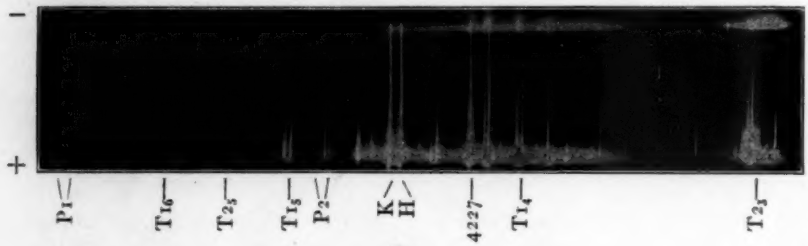


FIG. 3

To show these changes of intensity from pole to pole, the image of the arc was projected upon the slit of a quartz spectrograph and the intensity-gradients along the lines were observed as shown in Fig. 3. The scale of the quartz plates toward the red end of the spectrum was rather small, hence the arc was also photographed in the first order of a 30-foot Littrow spectrograph. These plates showed that intensity-gradients existed throughout the spectrum similar to those indicated by the quartz spectrograms for the violet region.

The pole-effects for the groups of the first and second subordinate series are collected in Table II.

It will be observed that these values for positive minus center are somewhat larger than those reported in a previous paper,[1] while the values for negative minus center are less. This difference is no doubt due to the use of the water-cooled arc in the present

[1] *Astrophysical Journal*, 43, 161, 1916.

investigation, since on account of the greatly increased steadiness it was possible to keep the slit of the spectrograph upon the bright spot at the pole of the arc. In this way a greater difference of intensity was introduced between the positive pole and the center of the arc and the greater pole-effect observed supports the hypothesis advanced in the previous article.

TABLE II

POLE-EFFECT FOR FIRST AND SECOND SUBORDINATE SERIES

Series	Pos.—Center	Neg.—Center	Series	Pos.—Center	Neg.—Center
$T1_4$.........	−0.003A	+0.001A	$T2_3$........	+0.017A	−0.001A
$T1_5$.........	− .037	+ .005	$T2_4$........	+ .024	− .003
$T1_6$.........	− .050	+ .011	$T2_5$........	+ .030	− .003
			$T2_6$........	+ .034	− .005

At the negative pole, furthermore, the photographs of the intensity-gradients, made in the larger scale of the Littrow spectrograph, showed that the intensity of these lines diminished continuously as the pole was approached, but that a small increase in intensity was experienced as the line traversed the bright spot which terminated the arc. In this case, then, the difference of intensity between the center of the arc and the negative pole was less and the corresponding pole-effect was also less than that observed before.

The relations between pole-effect and wave-length have been investigated for the first and second subordinate series, the narrow triplet series, the single-line series SL2, and also for three groups of so-called non-series lines which appear at λλ 6499–6449, λλ 5603–5582, and λλ 5270–5261. The average wave-length of a group, or triplet, was compared with the average total pole-effect or positive minus negative as obtained in Table I. The number of determinations available for any one sequence, however, was necessarily restricted and but little weight can be attached to the results obtained.

The data for the first subordinate series are of some interest, as Royds[1] has called attention to the peculiar appearance of the lines of

[1] *Ibid.*, 41, 154, 1915.

this series in the spectrum of barium together with the somewhat similar case for calcium, and from Table II it is seen that the pole-effect for the group $T1_4$ was small as compared with that observed for the other two groups of this series. This would seem to indicate, in accordance with Royds's suggestion, that the pole-effect for the group $T1_3$ in the infra-red might be of the opposite sign to that observed for the other members of the series. The difficulty attending the measurement of these very unsymmetrical lines is quite well known. The departure here, however, could hardly have resulted from the unsymmetrical widening which appeared with the fifth and sixth groups only, since the third group in the second subordinate series at λ 6160 was quite as symmetrical as the fourth group of the first subordinate series, but no similar peculiarity in the pole-effect and wave-length relations was observed. It is of interest to note in this connection that the lines in the group at λ 6160 were at times unsymmetrically reversed, but that whether reversed or not the measured pole-effects were identical.

The pole-effect and wave-length data for the second subordinate series were more systematic and indicated that here the pole-effect was inversely proportional to the cube root of the wave-length. This is shown by the curves of Fig. 4. The abscissae are wave-lengths and the ordinates for curve *a* are $1/P$, for curve *b*, $1/P^2$, for curve *c*, $1/P^3$, and for curve *d*, $1/P^4$, where P is the total pole-effect, or positive minus negative. The values of the ordinates were so adjusted that the first and last points on the four curves were coincident. It is of considerable interest to note that curve *c* intersects the wave-length axis at very near λ 2938, the convergence wave-length of the series. Swaim found the pressure-shift of the second subordinate series lines of zinc to be inversely proportional to the first power of the wave-length.

A similar treatment of the data for the non-series groups showed their pole-effect to be inversely proportional to about the seventh or eighth power of the wave-length, while that for the narrow triplet series and the series SL2 seemed to be inversely proportional to the fourth and fifth power of the wave-length. No great weight should be attached to these results, however, as the data are altogether too meager.

In conclusion it may be said that the additional data brought forward in this investigation confirm very closely the conclusions stated in the first paper, namely, that the pole-effect depends in general upon the existence of an intensity-gradient along the arc

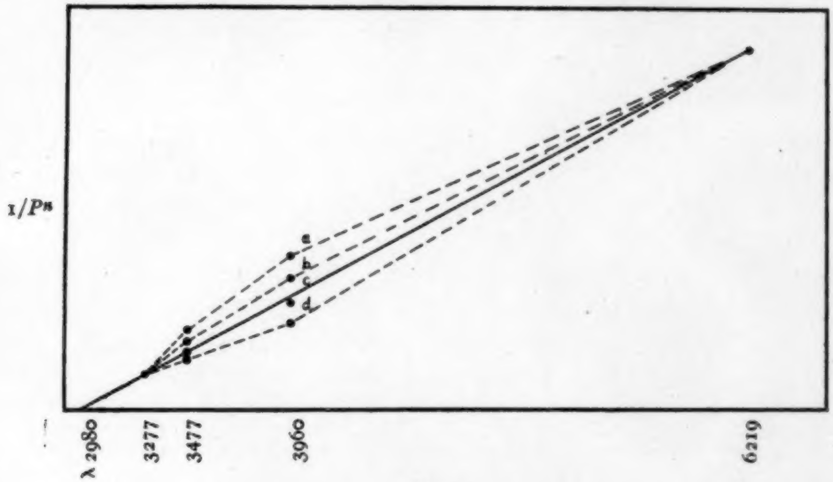


FIG. 4

and the hypothesis that the amplitude of vibration of the electrons is the determining factor in the pole-effect is thereby strengthened.

I wish here to express my thanks to the members of the Department of Physics and especially to Professor Gale, who suggested the problem, for his continued interest and assistance.

RYERSON PHYSICAL LABORATORY
June 1916

ON SPECTROSCOPIC RESOLVING POWER

By C. M. SPARROW

If a spectroscope is just able to separate two monochromatic lines of equal intensity and wave-lengths λ and $\lambda+\Delta\lambda$, the ratio $\frac{\lambda}{\Delta\lambda}$ is called the resolving power of the instrument for the wave-length λ. This is the *definition* of resolving power, and if we can determine by actual measurement the value of $\Delta\lambda$ for some particular instrument, we can obtain the resolving power of that instrument. If, however, our problem is to calculate the resolving power from the optical theory of the instrument, the *definition* must be supplemented by a *criterion* of some sort which will enable us to say when the two lines are to be considered as just resolved. In the case of a prism without absorption, or of a grating with many lines, the criterion proposed by Rayleigh[1] has hitherto been universally adopted. The intensity in a single line being given by

$$I=I_0\,\frac{\sin^2 x}{x^2}\,, \tag{1}$$

and that due to two lines by

$$I=I_0\left\{\frac{\sin^2(x-a)}{(x-a)^2}+\frac{\sin^2(x+a)}{(x+a)^2}\right\}, \tag{2}$$

the two lines are considered as just resolved when $a=\frac{\pi}{2}$, that is, when the maximum of one line coincides with the first minimum of the other. Under these conditions the composite diffraction pattern has a distribution of intensity given by the familiar curve 6 of Fig. 1. The ratio $\frac{I_{\min}}{I_{\max}}$ is in this case $\frac{8}{\pi^2}$ or about 0.81.

As originally proposed, the Rayleigh criterion was not intended as a measure of the actual limit of resolution, but rather as an index of the relative merit of different instruments. In the form in which it is stated above, the criterion is applicable only to instruments

[1] *Philosophical Magazine* (4), **47**, 193, 1874; (5), **9**, 266, 1879; also article on "Wave Theory" in the *Encyclopaedia Britannica*.

whose diffraction pattern is of the form (1). For such instruments it is as good an index as any other, and leads to simple formulae for the prism and grating. For instruments such as an absorbing prism or a Fabry and Perot interferometer it ceases to be immediately applicable. For such instruments we may, it is true, express the criterion in the form

$$\frac{I_{\min}}{I_{\max}}=0.81\,, \tag{3}$$

and this course has been generally adopted heretofore.[1] But now the criterion has lost its simple theoretical significance, and the choice of the value 0.81 for the right-hand side of (3) has become an arbitrary one. Moreover, the relative merit of different instruments will vary with our choice of the right-hand member of (3). Thus suppose that a grating and a Fabry and Perot interferometer have equal resolving power on the basis of (3). On a 90 per cent basis the interferometer would be superior, while on a 70 per cent basis the advantage would lie with the grating. If we should follow Schuster's proposal[2] and take complete separation as a basis, an infinitely thick prism with finite absorption would have zero resolving power.

It should be clear from the foregoing that the only fair basis on which such different instruments can be compared involves the adoption of a criterion which gives a measure of the actual limit of resolution. It has hitherto been assumed by many that the Rayleigh criterion does this, but the basis of fact on which this assumption rests is small and inconclusive; and, as we shall see, the true limit is quite different.

In the present paper we shall present the results of an empirical study of the actual appearance, visual or photographic, of different doublets. In this way the actual limits of resolution are determined. The results of these observations lead to the formulation of a criterion with a simple theoretical basis and applicable to a great variety of instruments. In addition the limits of resolution

[1] See, for example, Wadsworth, *Philosophical Magazine* (6), 5, 355, 1903, where the effect on resolving power of absorption in a prism system is calculated.

[2] *Theory of Optics* (London, 1909), p. 158.

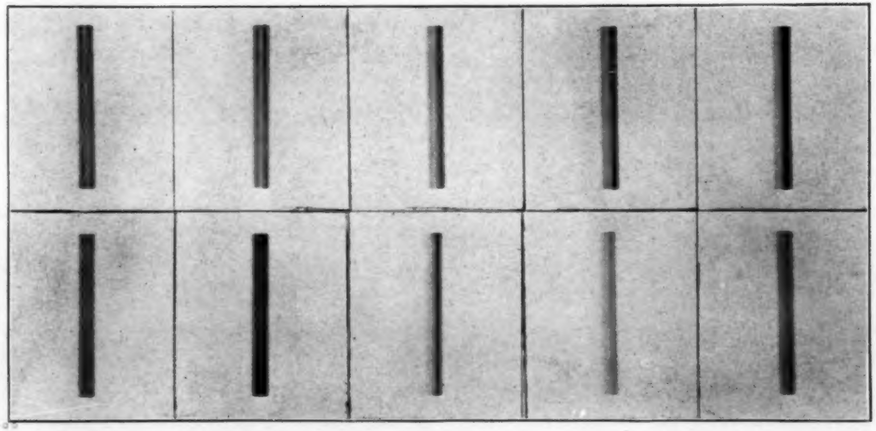
for two lines of different intensities have been determined—a case for which the theory gives no inkling as to what we may expect.

The experimental method is simple, and by no means new, being the same as that used by Langley for the conversion of bolographs into spectrographs. The form of a diffraction pattern was calculated and the intensity-curve, in rectangular co-ordinates, was drawn carefully on black paper. The area between the curve and the x-axis was then cut out, making a screen with an aperture of the required form. This screen was placed against a uniformly illuminated background and viewed or photographed with a cylindric lens having its axis of curvature parallel to the y-axis of the curve. In this way "artificial doublets" of any form and separation could be produced. The screens were about the size of a lantern slide, and the lens, of about 15 cm focus, was about 10 m from the screen. The camera was fitted with a multiplying back, so that six exposures could be taken on one plate. In order to test the focus, one exposure on each plate was made of a screen with a pair of narrow parallel slits. Hammer lantern plates (white label) were used; they were developed with hydrochinon. Visual observations on the lines led in all cases to the same results as the photographs; hence they are not specially mentioned in what follows.

Grating with infinitely narrow slit.—The actual appearance of a doublet whose separation is that given by the Rayleigh criterion is shown in the first row (1–5) of photographs in Plate II for different relative intensities of the two components. Considering for the moment only the case of equal intensity, it is obvious that the lines are quite distinctly resolved. On the plates the effect is so much more pronounced that most spectroscopists would call the separation measurable. The numbers which give the separation are the values of $2a$ in (2). They are thus half the phase difference in radians between the maximum of either line and the position on its diffraction pattern where the other maximum falls. The corresponding intensity-curves for each line are given in the first row of Fig. 1. In the second row of photographs (6–10) the components are closer, but are still clearly resolved. (The intensity-curves bear the corresponding numbers in Fig. 1.) As the lines are brought

PLATE II

1	2	3	4	5
$I_1=I_2$	$I_1=0.9I_2$	$I_1=0.7I_2$	$I_1=0.6I_2$	$I_1=0.5I_2$
$2a=\pi$	$2a=\pi$	$2a=\pi$	$2a=\pi$	$2a=\pi$



6	7	8	9	10
$I_1=I_2$	$I_1=0.9I_2$	$I_1=0.5I_2$	$I_1=0.7I_2$	$I_1=0.8I_2$
$2a=2.95$	$2a=2.95$	$2a=2.95$	$2a=2.9$	$2a=2.85$

SPECTROSCOPIC RESOLVING POWER

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still closer, the central minimum becomes shallower, until it finally disappears. To find the value of $2a$ corresponding to this condition

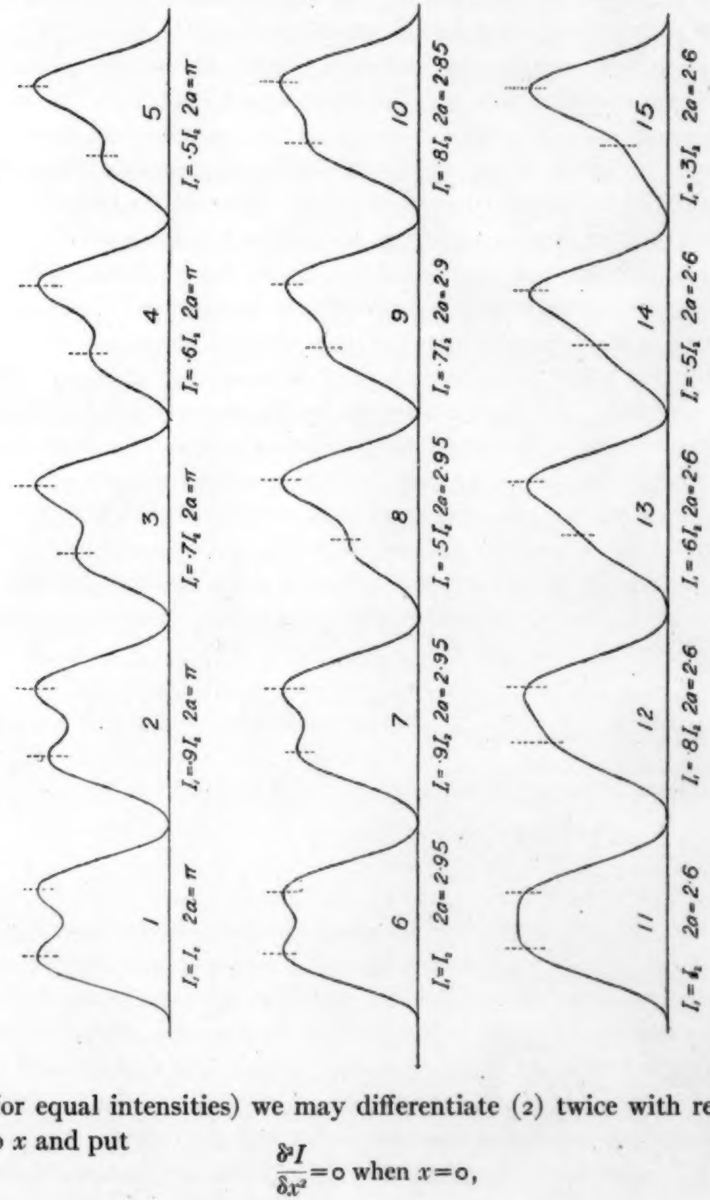


FIG. 1

(for equal intensities) we may differentiate (2) twice with respect to x and put

$$\frac{\delta^2 I}{\delta x^2}=0 \text{ when } x=0, \tag{4}$$

and solve for a. Since the two curves are symmetrical, the odd derivatives necessarily vanish at the origin, and thus (4) gives the composite curve an "undulation" at the origin. I shall refer to (4) hereafter as the "undulation condition."

It is obvious that the undulation condition should set an upper limit to the resolving power. The surprising fact is that this limit is *apparently actually attained*, and that the doublet still appears resolved, the effect of contrast so intensifying the edges that the eye supplies a minimum where none exists. The effect is observable both in positives and in negatives, as well as by direct vision. It cannot be seen in prints because of insufficient illumination. I have therefore not attempted to reproduce it here, but have given only the forms of the intensity-curves (11–15, Fig. 1). My own observations on this point have been checked by a number of my friends and colleagues. The same phenomenon has been noted by Wood in connection with the apparent reversal of a broad spectral line. A very slight further diminution of the separation rounds the top of the intensity-curve so that there is no resolution; the undulation condition thus defines the limit of resolution quite sharply.

A solution of (4) by successive approximations leads to the value $2a=2.606$. Thus the actual resolving power of a perfect grating of n lines in the Nth order is

$$\frac{\pi}{2.606}\cdot Nn=1.26\,Nn, \tag{5}$$

and similarly for a prism of thickness t and refractive index μ,

$$1.26\,t\frac{d\mu}{d\lambda}. \tag{6}$$

When the intensities are unequal, the form of the intensity-curve is of course completely altered. Nevertheless the actual observations show the remarkable fact that the *limit of resolution remains about the same*. In this case it is of course impossible to say definitely where resolution stops: a line which one observer would call resolved would perhaps be regarded by another observer as a single line shaded on one side. Nevertheless the form of the intensity-curve is quite sensitive to small changes in separation,

the result being that the actual limits given by different observers who have examined the plates vary by only a small percentage. When the ratio of intensities is greater than 10: 2 the appearance of the doublet is complicated by the greater relative prominence of the secondary maxima; it has therefore been found difficult to draw definite conclusions for intensity-ratios greater than this. The form of the intensity-curves is rather noteworthy; the curves, 8, 9, for example, show no actual minimum, but slope away continuously from the vertex, while those (11–15) which correspond to the limiting case hardly suggest a doublet by their shape.

Instruments with diffraction patterns other than the normal one.—Besides imperfections in the optical surfaces (which we shall not consider here) there are two principal causes for the deviation of the diffraction pattern from the form (1); namely, the use of a slit of finite width, and absorption or loss of light by reflection. The previous work with the normal pattern having shown that the resolving power varies at most only slightly with the relative intensity, it was found possible in subsequent work to simplify the experimental method. Instead of making a separate screen for each doublet (about a hundred such were used for the observations described above), one screen was made having the form of the diffraction pattern of a single line. This screen was mounted so that it could be displaced parallel to the x-axis through any required distance, and the doublets were made by superposing two exposures of the screen in different positions on the same plate, intensities being regulated by the time of exposure.

Two cases were studied in detail: that of a grating with "4-normal" slit-width, and that of an infinitely thick prism with finite absorption. Detached instances were studied for other cases. The general results may be summed up by the same criterion as that found for the grating with a narrow slit; namely, that *the limit of resolution is given by the undulation condition.* Since this was found to hold for a narrow slit and a wide slit, it seemed safe to assume that it would hold for intermediate slit-widths. Since it holds for an infinitely thick absorbing prism and for a perfectly transparent prism, it may be assumed to hold for all cases which are intermediate between these two, or which approximate them very closely.

As will be seen below, these cases include a finite absorbing prism, an echelon grating, a Lummer-Gehrcke plate, and a Fabry and Perot interferometer. As this list includes most of the important forms of spectroscopic apparatus, it may be concluded that the undulation condition furnishes a criterion of very general applicability. There thus remains only the task of formulating this criterion for the different types of instrument. This formulation is best expressed by the use of factors which indicate the relative resolving power of such instruments with respect to the more perfect instruments. We thus have two sets of factors, slit-width factors and absorption factors. In order to calculate, for instance, the resolving power of an echelon grating, taking account of absorption and slit-width, we have only to multiply the resolving power for the ideal instrument by a suitable factor which depends only on the absorption and slit-width, not on the type of instrument.

The slit-width factors.—The intensity pattern of a doublet may be written in the form

$$I=I_0\left\{\int_{x-d}^{x+d}\frac{\sin^2(x+a)}{(x+a)^2}\,dx+\int_{x-d}^{x+d}\frac{\sin^2(x-a)}{(x-a)^2}\,dx\right\}. \qquad (7)$$

The analytic expression for the undulation condition is not in this case easy to apply. The values of $2a$ were therefore obtained by a

TABLE I

Slit-Width $\times f/d$	$2a$	Slit-Width Factor (C.M.S.)	Purity Factor (Schuster)
0.	2.606	1.00	1.00
0.25.	2.64	0.99	0.986
0.5.	2.72	0.96	0.943
0.75.	2.91	0.90	. . . .
1.00.	3.14	0.83	0.780
1.25.	3.77	0.69	. . . .
1.50.	4.75	0.55	0.579
1.75.	5.61	0.46	. . . .
2.00.	6.30	0.41	0.450

combination of graphical and numerical methods. The results are given in Table I. The "purity factors" of Schuster,[1] which

[1] *Astrophysical Journal*, 21, 197, 1905; *Theory of Optics* (London, 1909), p. 163.

were calculated for the same purpose, but with a different theoretical basis, are given for comparison in the fourth column. The difference in the two sets of factors is not great, and either would probably prove sufficiently accurate for most practical purposes.

Absorption factors.—The problem of finding the diffraction pattern is here one of combining n disturbances with amplitudes in geometric progression and phases in arithmetic progression. The summation leads to the well-known formula of Airy which we may write in the form

$$I = s_0^2 \frac{1-2r^n \cos n\phi + r^{2n}}{1-2r \cos \phi + r^2}, \tag{8}$$

where r is the ratio of the $(p+1)$th to the pth amplitude, s_0 the initial amplitude, and ϕ the phase difference between successive disturbances. The equation which expresses the undulation condition is here quite complicated unless n is infinite. For most practical purposes an approximate formula will do as well. We may obtain such a formula by making n infinite, while the total phase change $n\phi$ and the total absorption r^n, as well as ns_0, approach finite limits. Writing $r=e^{-k}$ and multiplying and dividing numerator and denominator in (8) by n^2,

$$I = \frac{n^2 s_0^2 (1-2e^{-nk} \cos n\phi + e^{-2nk})}{n^2(1-e^{-k})^2 + 4e^{-k} \cdot n^2 \sin^2\frac{\phi}{2}} \tag{9}$$

$$= I_0 \frac{(1-2e^{-k} \cos \Phi + e^{-2k})}{k^2 + \Phi^2}. \tag{10}$$

Here I_0 is the maximum intensity which we should have without absorption, Φ is the total phase difference between the two extreme disturbances, and k is the logarithm of the ratio of the final to the initial disturbance. The approximation amounts to this: if we represent the disturbances by vectors, the vector sum (9) is a polygon inscribed in a logarithmic spiral; in (10) we pass from the polygon to the limiting spiral. Equation (10) is a rigorous expression for an absorbing prism[1] and an approximate one for the

[1] See Wadsworth, *op. cit.*, ... , where essentially the same formula is derived.

case of an echelon grating, or a Lummer-Gehrcke plate. To form some idea of the degree of approximation the value of $2a$ for $n=5$, $k=1$ was computed by both (8) and (10). The value from the exact formula was about 2 per cent greater than that from the approximate formula. As this value of n is very small for an actual instrument, and as the accuracy of (10) increases very rapidly with increasing n, we may consider (10) a sufficient approximation for most purposes.

The undulation condition for equation (10) was solved for different values of k by successive approximations, giving the absorption factors listed in Table II. The first column gives the values of k, the second the corresponding values of $e^{-k}(=r^n$, see (8)), the third gives the values of $2a$, and the fourth the absorption factors.

TABLE II

k	e^{-k}	$2a$	Absorption Factor
0	1.0000	2.606	1.00
0.5	0.6065	2.611	0.998
1.0	0.3679	2.637	0.988
1.5	0.2231	2.662	0.979
2.0	0.1353	2.710	0.962
4.0	0.0183	3.041	0.857

For infinite values of k the expression (10) becomes indeterminate, since I_0 also becomes infinite. By returning to (9) we may obtain an expression for the intensity in this case, which is the case of an infinitely thick prism with finite absorption. The expression here reduces to the simple form

$$I=\frac{I_1}{\Phi_1^2+k_1^2}, \tag{11}$$

where I_1, k_1, and Φ_1 have the same meaning as the corresponding quantities in (10) except that they refer to any finite portion of the prism. If we make I_1 the intensity of the incident light, k_1 the logarithmic decrement due to loss by reflection and absorption, and Φ_1 the phase difference between two successive interfering beams, the expression (11) is an approximate expression for the intensity-curve of the Fabry and Perot interferometer. The undu-

lation condition obtained from (11) leads to an extremely simple formula for the resolving power of this instrument. Writing

$$I=\frac{s_1^2}{k_1^2+(\Phi_1+\Phi_0)^2}+\frac{s_1^2}{k_1^2+(\Phi_1-\Phi_0)^2},$$

differentiating twice as to Φ_1 and putting $\Phi_1=0$, we obtain

$$4\Phi_0^2=k_1^2+\Phi_0^2 \text{ or } \Phi_0=\frac{k_1}{\sqrt{3}}. \tag{12}$$

If D is the distance between the plates, this gives for the resolving power

$$\frac{2\pi D}{\lambda}\frac{\sqrt{3}}{k_1}=\frac{10.9\,D}{\lambda k_1}. \tag{13}$$

It is worth while to compare this result with that obtained from the exact formula, which may be obtained from (8) by making n infinite. The undulation condition leads to a quadratic in $\cos\Phi$, the solution of which gives

$$\cos\Phi_0=-\frac{1+r^2}{4r}+\sqrt{\frac{(1+r^2)^2}{16r^2}+2}\quad (r=e^{-k_1}). \tag{14}$$

For $k_1=0.1$ we obtain from (12) $\Phi_0=0.1155$, and from (14) $\Phi_0=0.1158$, thus showing that (11) represents the form of the Fabry and Perot fringes in the neighborhood of a maximum with a high degree of approximation.

There is one further advantage of the criterion furnished by the undulation condition, namely, that it is independent of any particular photographic process; for contrast can be enhanced by photography only where it exists, so that we should expect the appearance of a pair of lines at the limiting separation to undergo little change with any variation of the photographic process.

Visual resolving power.—It is obvious that the undulation criterion should apply equally to the calculation of the visual (telescopic) resolving power of a rectangular aperture. For apertures of other shapes we should not a priori expect it to apply. The problems presented are of far less practical importance than those furnished by the spectroscopic case, and it has not seemed worth while to carry the investigation farther in this direction.

SUMMARY

1. The actual limit of the resolving power of a perfect grating or prism has been determined experimentally. It is found that this limit is given, for equal intensities of the two lines, by the "undulation condition," that is, by the condition that the central minimum shall just disappear. This gives a theoretical resolving power about 26 per cent greater than that obtained by the Rayleigh criterion.

2. The limit given by the undulation condition has been found to hold for unsymmetrical doublets when the ratio of intensities of the two components is less than 10:3.

3. The undulation condition gives the limit for all cases in which the diffraction pattern is modified by finite slit-width, or by a decrease in geometric progression of the intensities of the interfering beams, whether this is due to absorption or to loss of light by reflection. These cases include most of the important forms of spectroscopic apparatus.

4. The effect of slit-width and absorption can be introduced by the use of suitable factors. These factors have been calculated for various values of the slit-width and absorption.

5. A simple approximate formula has been given for the resolving power of the Fabry and Perot interferometer.

The foregoing work was begun during the last Christmas vacation in the Physical Laboratory of the Johns Hopkins University. I am indebted to the Department of Physics there for the facilities so freely placed at my disposal during the beginning of the work, and for the loan of the cylindric lens with which I have continued the work here. I am also especially indebted to Dr. J. A. Anderson for his valuable advice and assistance.

Rouss Physical Laboratory
University of Virginia
July 1916

THE EFFECT OF AN ELECTRIC FIELD ON THE LINES OF CALCIUM AND LITHIUM[1]

By JANET T. HOWELL

INTRODUCTION

After the discovery of the Zeeman effect the analogous decomposition of spectral lines in an electric field was looked for by many investigators. In 1913 the effect was discovered in the diffuse series of hydrogen by J. Stark[2] and A. Lo Surdo[3] under entirely different experimental conditions. Since then it has been studied in hydrogen and helium by both methods and Stark has investigated the transverse effect for lithium, mercury, and a number of other elements.

Although a very large number of data have been accumulated since the discovery of the new effect, the work in this important field is still in its infancy. The results obtained by Stark and Lo Surdo differ markedly for the elements investigated, and the number as yet uninvestigated is large. The results have a most important bearing on atomic theory,[4] but, so far, they seem to have led to new complications rather than to new solutions. The contradictions between the results given by Stark and Lo Surdo suggest the possibility that the electric effect may vary under different conditions, but no definite conclusion on this subject has been reached. To make a comprehensive theory of the effect possible, much more investigation is necessary. Both the methods employed thus far have serious limitations, hence new ways of attacking the problem are important. It is especially important, from the point of view of solar work, to find a method adapted to the investigation of the heavy elements of the reversing layer. Although no new

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 121.

[2] *Berichte der K. Preuss. Akad. der Wiss.*, **47**, 932, 1913.

[3] *Rendiconti d. Lincei*, **22**, 2d sem., 664, 1913.

[4] A. Garbasso, *ibid.*, **22**, 2d sem., 635, 1913; *Il Nuovo Cimento*, **7**, 354, 1914; *ibid.*, **9**, 376, 1915. W. Wien, *Berichte der K. Preuss. Akad. der Wiss.*, **48**, 70, 1914; W. Voigt, *Annalen der Physik*, **4**, 197, 1901; *Göttingen Nachr. Ges. Wiss.*, 1914; E. Gehrcke, *Verh. der deutschen phys. Ges.*, **16**, 431, 1914; N. Bohr, *Phil. Mag.*, **27**, 506, 1914.

method has been perfected as yet, a preliminary survey of a large number of elements under low dispersion has brought out some new and interesting results in calcium and lithium which are reported here.

The results obtained in previous work can be summarized briefly as follows.

STARK'S METHOD AND RESULTS

The great number of data contributed by Stark and his co-workers have been collected and discussed in Stark's *Elektrische Spektralanalyse chemischer Atome.*[1] They used the luminous canal rays behind the cathode in a discharge tube as the source of light, and submitted them to an auxiliary electric field of 28,500–74,000 volts per centimeter. The source of potential for this auxiliary field was a dynamo of 4,500 volts and a storage battery of 3800 volts. In his earlier work Stark used a small concave grating of 1.5 m radius giving a dispersion in the first order of 1 mm = 9 A. Later, in photographing the fine division of the hydrogen lines, he decreased the dispersion somewhat, but more than doubled the electric field. He used two methods for obtaining the spectra of elements in the electric field: (1) a luminous gas, (2) the bombardment of the salts of alkali metals by the canal rays. He also mentioned the possibility of producing metallic lines by the bombardment of metal electrodes by canal rays.

Stark investigated the transverse effect for H, He, Li, Hg, Al, C, Ca, Mg, Na, and Th, and the longitudinal effect for H and He.[2] In both cases he observed in a direction perpendicular to the direction of motion of the canal rays in order to avoid complications due to the Doppler effect. The general conclusions derived from his results have been summarized by Fulcher[3] and may be put briefly as follows:

1. The diffuse series of H, He, and Li show a separation directly proportional to the field-intensity and of an order of magnitude of 3–18 A for a field of 28,500 volts per cm.

[1] Leipzig: S. Hirzel, 1914.

[2] *Annalen der Physik*, **43**, 965, 1914; J. Stark and G. Wendt, *ibid.*, **43**, 983, 1914; J. Stark and H. Kirschbaum, *ibid.*, **43**, 991, 1017, 1914; J. Stark, *ibid.*, **48**, 193, 1915.

[3] *Astrophysical Journal*, **41**, 359, 1915.

2. The hydrogen components are symmetrical as to the displacement from the original line and probably as to intensity in sources at rest. The red components are more intense when the field and the motion of the luminous particles are in the same direction, and the violet components when they are in the opposite direction.

3. Helium and lithium show asymmetry both as to intensity and as to displacement.

4. Unlike the magnetic effect, the electric effect varies from line to line of the same series. The number of components and the maximum displacement increase with the term-number.

5. The diffuse series shows large effects, while the separation in the sharp main and subordinate series of He and Li is less than 1 A except for the He lines λ 3613.8 and λ 4169.1.

6. Lines of Al, C, Ca, Hg, Mg, Na, and Th show practically no effect. The displacement in the diffuse series of Hg is the largest, being of the order of magnitude of 0.4 A.

7. In the transverse effect the components are polarized, the outer lines parallel to the field, the inner perpendicular.

8. In the longitudinal effect of hydrogen and helium the lines are unpolarized and agree in number and position with the perpendicular components of the transverse effect.

LO SURDO'S METHOD AND RESULTS

Lo Surdo observed the region immediately in front of the cathode in a discharge tube, where the luminosity of the negative glow and the sudden fall of potential fulfilled the conditions for electric decomposition. Investigating the conditions in the dark space,[1] he showed that the increase in the cathode fall at a certain pressure depends solely on the current-density. He therefore used very narrow tubes. He found, experimentally, that when the plain electrode completely fills the tube, the length of the dark space is independent of the diameter of the tube. It is essential, in any case, to have the electrode completely fill the tube so that the lines of force may remain parallel.

[1] *Il Nuovo Cimento*, 9, 368, 1915.

In his earlier work Lo Surdo used a tube of 4 mm internal diameter and 20 cm long, but later decreased the diameter to 1.5 mm. The pressure was regulated in general to a 2 mm dark space and the tube was excited by storage batteries giving a potential difference of 5000–8000 volts. The image of the region in front of the cathode was polarized by a nicol and focused on the slit of a 4-prism spectroscope. In the transverse effect the varying electric field gives the lines the shape of a Y, making it easy to identify the components.

The hydrogen results given by Lo Surdo[1] and Puccianti[2] differ somewhat from those of Stark. Lo Surdo finds two parallel components in all the hydrogen lines investigated (α, β, γ, δ) and perpendicular components agreeing in number with the term-number of the series. Stark, even in his early papers, found more components than this, and his latest work shows the hydrogen lines to be very complex. Lo Surdo's examination of the longitudinal effect in $H\gamma$ agrees with Stark's result.

Brunetti[3] has recently published two papers on the electrical decomposition of helium lines. The work shows the presence of very interesting satellites which follow a different field law from the regular components. The components are, in general, somewhat different from Stark's, especially as regards polarization.

APPARATUS

The apparatus used in this work was essentially of the Lo Surdo form. Professor Stark was kind enough to prepare for the Mount Wilson Observatory a hydrogen tube, of the ingenious form devised by him. But as the life of such tubes is short, and as no glassblower having sufficient skill to make others was available, we were compelled to have recourse to the very simple form of tube used by Lo Surdo. Moreover, Stark has made a survey of most of the more promising elements with his apparatus, but the Lo Surdo tube has been applied only to hydrogen and helium. The discordant results of the two schools of investigators seemed to indicate that the

[1] *Rendiconti d. Lincei*, **23**, 1st sem., 82, 143, 252, 326, 1914.

[2] *Ibid.*, pp. 329, 331, 1914.

[3] *Il Nuovo Cimento*, **10**, 34, 41, 1915.

nature of the Stark effect is dependent on the mode of excitation. So the application of the Lo Surdo method to all the available elements was a hopeful point of departure. A diagram of the apparatus is shown in Fig. 1.

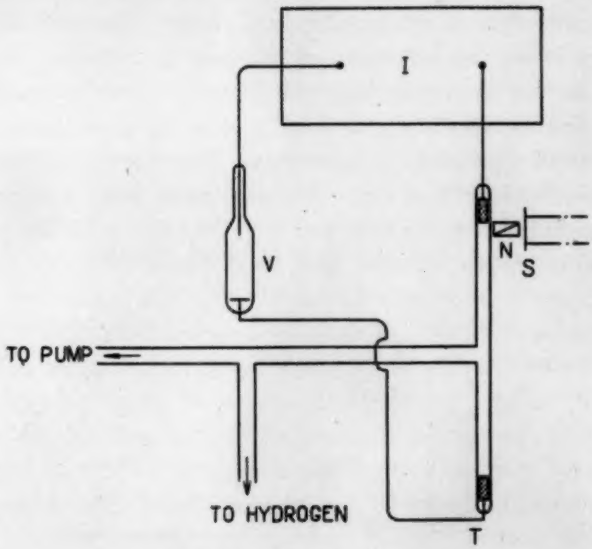


FIG. 1

T = Tube
V = Valve tube for rectifying the discharge
I = Induction coil
N = Nicol
S = Slit of 3-prism spectroscope

The spectroscope contained three prisms, two of ultra-violet glass, one of quartz (compound right- and left-handed), and gave a dispersion of 1 mm = 18 A at Hγ; 1 mm = 12 A at H and K. The source of light was placed directly in front of the slit with a small nicol interposed. This arrangement introduced some astigmatism, but, as only the maximum displacement was measured, the integrated effect along the line did not interfere. As the collimator was entirely filled, the arrangement was economical of light and also of space. An oil pump was used which held the pressure steadily at about 6 mm dark space. No value can be given for the current through the tube as no suitable instrument was available

for measuring it. The current in the primary of the coil was 11 amperes, the voltage 110, and a Wehnelt interrupter was used for which the maximum spark-gap of the coil was 17 cm. A valve tube in the circuit rectified the discharge.

As the tube cracked on overheating, it was run intermittently and not more than three minutes at a time. The interval covered by the exposure varied from 30 minutes to 6 hours, the actual exposure being about one-half this time. Seed "27" plates were used. The tube was a simple T-tube of heavy-walled glass tubing with electrodes sealed in at the ends. The internal diameter was 6 mm and the length 20 cm. The electrodes were solid cylinders of metal nearly filling the tube and were sealed in with fine platinum wires. Three types of tubes were used whose cathodes are shown in Fig. 2.

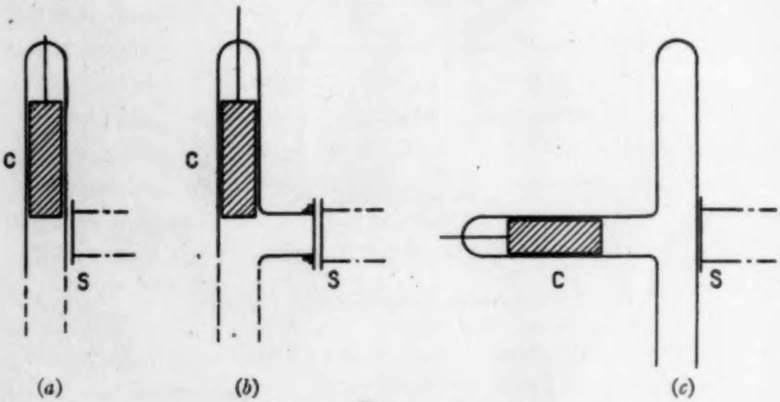


FIG. 2

S = Slit of spectroscope
C = Cathode

Types (*a*) and (*b*) were used in the transverse effect. When the electrodes were of aluminium, which did not sputter to any extent during the life of the tube, the simple form (*a*) was used. When the cathode was made of a metal that sputtered easily, or when a salt was used on the cathode that discolored the sides of the tube, type (*b*) was found to be very satisfactory indeed. Type (*c*) was used in the longitudinal effect and worked well except for its extreme fragility. Types (*a*) and (*b*) lasted for several days, if

used with care and caution, but in (*c*) the impact of the rays melted the glass directly opposite the cathode and the life of the tube was only from 30 minutes to an hour. A new tube could be made and set up in 20 minutes, but the fragility seriously limited the length of exposure possible. If any further work is done with tubes of this type it would be well to make them of quartz.

Photographs of the transverse effect showed electrical components in the Y-shape described by Lo Surdo. In the longitudinal effect the electrode did not entirely fill the tube, with the result that the field fell off at the edges and the components tapered inward and were easily identified.

METHODS OF OBTAINING SPECTRA

The discharge tube was always filled with hydrogen so as to have standard lines for field determination. The metallic cathode, bombarded by anode rays, gave the spectrum of the metal close to its surface, in just the proper region for observing the electric effect. This was true of all the metals investigated. Stark mentioned this method of obtaining spectra in a vacuum tube. Goldstein[1] investigated the phenomenon and found that the spectra appeared only when the gas in the tube was nitrogen, and were much strengthened at a liquid air temperature. Robinson[2] recently published a paper on the cathode spectra of metals. He was able to obtain spectra in H, CO, and, with special brilliance, in O, but found it necessary to have thin cathodes. No evidence of metallic lines was obtained with electrodes 1–2 mm thick. Since I had no difficulty in producing brilliant metallic spectra near the cathode with cylindrical electrodes of 5 mm diameter and from 1.5 to 2 cm long, it follows that the thin electrode is certainly not necessary. The spectra were obtained with about equal brilliance in air, oxygen, and hydrogen. Comparative photographs were taken of the cathode spectra of aluminium and iron at different pressures and there seems to be no particular connection between the cathode spectra and the sputtering of the metal. Although iron sputters easily, and aluminium almost none at all, the aluminium spectrum

[1] *Physikalische Zeitschrift*, **6**, 14, 1905.

[2] *Astrophysical Journal*, **42**, 473, 1915.

was obtained over a larger range of pressure than the iron spectrum. The aluminium lines λ 3962 and λ 3944 appeared before the pressure was low enough for the Crookes's dark space to appear and at 3–4 mm dark space they extended 3 cm from the electrode. The iron spectrum does not appear until the pressure is reduced to 3 mm dark space. The spectra of Fe, Ni, Al, Mg, Zn, and Ca were obtained by this method and, as found by Robinson, they were in general like the spark. The metallic lines are easily identified, as they are more intense near the cathode, and it seems likely that further study of them will bring out many interesting differences from arc and spark spectra. So far the only one that has been studied in detail is the copper[1] spectrum. In the cathode spectrum of magnesium there were three lines which clearly originate in the electrode and which do not agree with any known magnesium lines. The wave-lengths are 4155, 4113, and 4103. The last is probably the same as λ 4106.8 given by Fowler,[2] but the other two seem to be new.

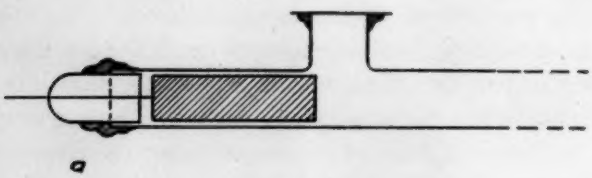


FIG. 3

When the cathode was covered with a thin coat of the chloride of a metal, carefully dried, the bombardment of the salt by the anode rays gave a very brilliant spectrum. Li, Ca, and Sr were used in this way. When using salts, the (*b*) or (*c*) type of tube was found best because the walls became discolored very fast. In order to renew the supply of salt between exposures the electrodes were made demountable as shown in Fig. 3. The electrode was sealed in at *a* with sealing wax.

METHOD OF MEASURING PLATES

The hydrogen lines Hβ and Hγ were used to determine the field-strength for each plate. Hβ gave a very clear separation when

[1] *Astrophysical Journal*, 42, 473, 1915. [2] *Proc. R. S.*, 71, 419, 1903.

polarized parallel, but was too indistinct for measurement when polarized perpendicular to the field. Hence for these plates only Hγ could be used. The red component of Hγ was usually confused by an air line. On some plates this line was absent, but whenever it was present only the violet component was measured. The field-strength was determined from Stark's measurements for the separation at 28,500 volts per centimeter. His later work showed the hydrogen separation to be symmetrical, and he claimed much greater accuracy for the total separation than for the displacement of the components from the center given in his earlier paper. It seemed best, therefore, to multiply the displacement of the violet component by two and use this value for the total separation when the red component could not be measured. A certain amount of inaccuracy is introduced here because the intensity of the components is slightly unsymmetrical for moving sources and this might shift the center of the middle component to one side. As no shift could be detected in the middle component, the error is apparently less than the error of measurement. There is, however, a large error in this method of determining the field-strength. Stark thought that his calculation of the field-strength from the potential difference and the distance between the electrodes was probably too large; hence the resultant separations for a given field-strength were probably too small. Also there is great likelihood that the magnitude of the separation may vary somewhat in going from the Stark to the Lo Surdo method where the conditions are different. This seems especially likely, since the field-strength often differed by 15 or 20 per cent in determinations from Hβ and Hγ on the same plate, while the determinations from Hγ on different plates, taken under the same circumstances, rarely varied more than 5 per cent. The general character of the components agreed with Stark's results, except that a single center component was always found in Hγ when polarized parallel, instead of the two faint ones given by Stark. This is probably due to the fact that the discharge was not absolutely unidirectional, and that a faint unseparated line is superimposed on the components. This indirect process of determining field-strengths undoubtedly introduces an error that may be as great as 10 per cent, but no better method was available.

Another uncertainty lies in the possibility of a complication from the Doppler effect when photographs are taken along the field. It is quite possible that this may have affected the measurements on Hγ. The center line of Hγ in the longitudinal effect is composed mainly of light from in front of the dark space and appears very sharp; consequently there is certainly no Doppler effect present there. The components, however, are composed almost entirely of light from the negative glow on the surface of the cathode where the Doppler effect would be at a maximum. However, Stark always found an undisplaced line as well as a displaced line in his photographs of the Doppler effect in canal rays, and the displaced line varied in displacement and intensity with the velocity of the rays. At the presssure used in these experiments it seems safe to assume that the displaced line, if present, would be very faint and very close to the stationary one. As the components of Hγ are themselves faint, only the stationary line would show. Lo Surdo photographed the longitudinal effect for Hγ and found symmetrical unpolarized components agreeing in position and number with those observed by Stark. Apparently, therefore, there was no Doppler effect noticeable in his case. H and K and Li λ 4602 give components strong enough to be confused by a Doppler effect, but the components do not show two maxima of intensity. At any rate, these lines have not thus far been examined in canal rays for a Doppler displacement; nothing definite, therefore, can be said about them in this respect.

The maximum separation of components was selected for measurement and their displacement from the center was determined by reference to a neighboring unaffected line and to the unseparated line outside the field.

The accuracy of the results varies with the dispersion, and the character of the lines. Some idea of the degree of accuracy will be given under the results on each line.

RESULTS

Entirely negative results were found for Fe, Ni, Al, Mg, Zn, and Sr.

The aluminium lines λ 3962 and λ 3944 were very greatly strengthened toward the cathode—more than the increase in

intensity at the cathode could account for. Photographs of these lines with an echelon showed wings to the red, but they did not vary with the field-strength. The aluminium spectrum was examined from λ 4860 to λ 2350 with a single quartz prism spectroscope, but no electric effect was found. In Mg, Fe, Ni, Zn, and Sr the spectra were photographed from λ 4900 to λ 3500, and no electric effect large enough to be detected with the low dispersion used was observed.

TABLE I

LITHIUM

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER (HOWELL)

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+1.00	8	+0.48	8	
	−2.48	6	−2.00	6	
4132.93.......	+2.26	2	+1.78	2	
	−0.18	5	−0.18	5	Unpolarized
	−3.10	1	−2.24	1	

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER (STARK)

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+0.65	8	+0.56	8	
	−0.28	1	−0.28	1	Doubtful
	−1.81	6	−1.64	6	
4132.93.......	+1.87	3	+1.57	3	
	−0.37	4	−0.33	4	
	−2.62	2	−2.44	2	

LONGITUDINAL EFFECT FOR 20,000 VOLTS PER CENTIMETER (HOWELL)

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+0.57	8	+0.34	6	
	−2.01	6	−1.53	3	
4132.93.......	+1.16	1	+0.77	1	
	−0.26	5	−0.26	5	Unpolarized
	−1.99	0	−1.50	0	

Results for lithium.—The results for lithium are given in Table I, together with Stark's results reduced to the same field-strength. The numbers marked + indicate components to the red, those marked −, components to the violet. The field close to the cathode was in the neighborhood of 25,000 volts per centimeter. In the

longitudinal effect, the displacement of the red component in λ 4602 was doubled because the photographed line was a blend of the red component and the unaffected line outside the field. For the same reason the displacement of the center component to the violet in λ 4132 (longitudinal) was doubled. As the discharge was not entirely unidirectional there is always a faint unseparated line superimposed on the electrical components, and this makes the displacement of the center component of λ 4132 too small in the transverse effect. But, as it was uncertain what percentage of the total intensity was due to this unseparated line, the measured displacement is given. Owing to the uncertain nature of the middle component of λ 4132 the total separation of the outer components is more accurate than their displacements from the center. The results for the transverse effect are in general larger than those found by Stark. As the error of the field-strength determinations is uncertain, the only idea of the precision of the results that can be given is the average deviation from the mean of the results on different plates. This deviation is given in Table II. The components of Hγ are more distinct when polarized perpendicular than when polarized parallel; consequently the error is greater for the parallel components. The deviation of λ 4602 refers to the total separation, that of λ 4132 to the separate components. Photographs of the lithium lines are given in Plate III.

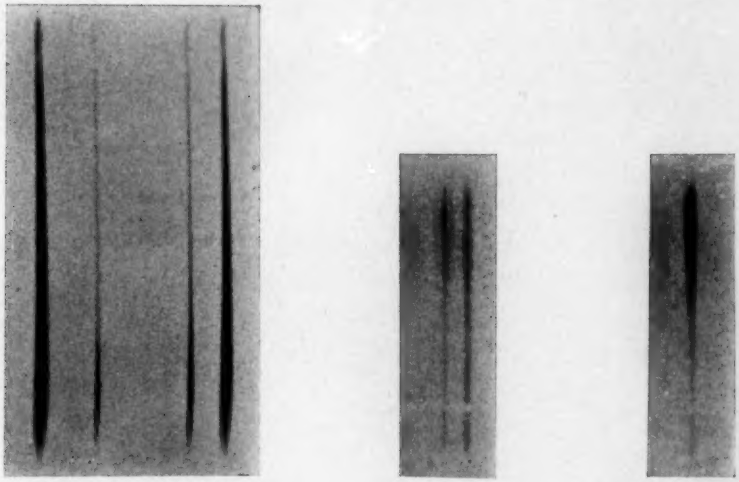
TABLE II

AVERAGE DEVIATION IN A FOR LITHIUM RESULTS

λ	Transverse Effect		Longitudinal Effect	
	Parallel Comp.	Perpend. Comp.	Parallel Comp.	Perpend. Comp.
4602.37......	0.32	0.04	0.14	0.04
4132.93......	0.28	0.02	0.12	0.12

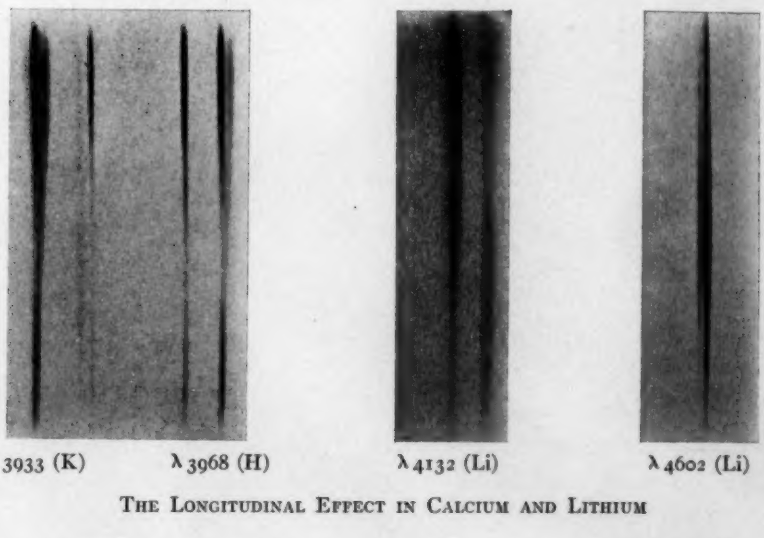
Results for calcium.—The results for the H and K lines of calcium are given in Table III. In the earlier photographs a cathode of metallic calcium was used, but it had a decided tendency to rectify the discharge in the opposite direction and acted as an

PLATE III



λ 3933 (K) λ 3968 (H) λ 4132 (Li) λ 4602 (Li)

The Transverse Effect in Calcium and Lithium



λ 3933 (K) λ 3968 (H) λ 4132 (Li) λ 4602 (Li)

The Longitudinal Effect in Calcium and Lithium

NU

anode such a large part of the time that the photographs were very poor. Later, calcium chloride on an aluminium cathode was used and proved very satisfactory. The longitudinal effect gave very clear components, but in the transverse effect the lines were both broad and hazy, although the Y-shape proved the effect to be due to the electric field. The best photographs showed the lines to be composed of two very diffuse components, an intense one slightly displaced to the red, and a fainter one more displaced to the violet.

Table III contains measurements on the separation of these very indistinct components in the transverse effect. The actual separation of the components is small, but the width of the lines, owing to their diffuse character, is large. As the red component is more intense and more diffuse than the violet, the width is almost symmetrical with respect to the unseparated line. The total width of H is 2.38 A parallel and 2.16 A perpendicular; that of K is 2.64 A parallel and 2.42 A perpendicular.

TABLE III

CALCIUM H AND K

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
3968.63.......	+0.22	6	+0.16	6	Unpolarized
	−0.86	2	−0.74	2	
3933.83.......	+0.22	9	+0.22	9	Unpolarized
	−0.92	3	−0.74	3	

LONGITUDINAL EFFECT FOR 20,000 VOLTS PER CENTIMETER

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
3968.63.......	+1.27	3	+1.23	3	Unpolarized
	+0.01	8	−0.02	8	Unpolarized
	−1.17	0	−1.11	0	Unpolarized
3933.83.......	+1.42	4	+1.38	4	Unpolarized
	+0.06	9	−0.02	9	Unpolarized
	−1.30	1	−1.26	1	Unpolarized

The results of the transverse effect indicate pretty clearly that H and K are both composed of a strong unpolarized component 0.2 A to the red and of a fainter polarized component to the violet. The

longitudinal components are certainly unpolarized. The middle line shows evidence of polarization on the edges, but the polarized edges are such a small part of the total intensity of the line that the effect is scarcely measurable. The H and K lines were investigated for circular polarization, but no trace of it was found.

Photographs of H and K are shown in Plate III. The separation of H and K into components in the transverse photograph does not show in the reproduction, but the progressive widening with increasing field-strength proves the electrical nature of the effect. The violet component of H in the longitudinal photograph does not reproduce well, but is analogous to that of K which shows plainly. The Y-shape in the longitudinal components means that the discharge was passing from only a small section of the electrode. This happened frequently when calcium chloride was used. When the whole surface of the electrode was acting, the components extended across the field, tapering in at the edges.

The average deviation of the results is given in Table IV. It is calculated for the total separation in the transverse effect and for the separation of the components in the longitudinal.

TABLE IV

Deviation in A for H and K Components

λ	Transverse Effect		Longitudinal Effect	
	Parallel Comp.	Perpendicular Comp.	Red Comp.	Violet Comp.
3968.63......	0.12	0.05	0.04	0.03
3933.83......	0.10	0.09	0.05	0.01

DISCUSSION OF RESULTS

Instead of contributing new law and regularity to the electric decomposition of spectral lines, the results for lithium and calcium seem to contradict the laws already formulated. In former investigations the longitudinal effect has given only unpolarized components, but the lithium lines λ 4602 and λ 4132 are here shown to have very clearly polarized components along the field.

Previously only the diffuse series of different elements have shown any appreciable effect, but H and K belong to a principal-pair series of calcium, while the lines of the diffuse series at $\lambda\lambda$ 4457, 4435, and 4425 showed no effect at all. The lines at λ 3737 and λ 3706, which are analogous to H and K, were too faint on my plates for any examination, but I hope to investigate them in the near future.

It is still somewhat uncertain whether the electric effect is the same under different conditions or not. The difficulty in determining consistent field-strengths from the hydrogen lines is no very strong evidence on either side, because the difficulty of making accurate measurements on the components of $H\gamma$ was so great. The values found here for lithium in the transverse effect are larger than Stark's values, but the difference is not great enough to show any real dissimilarity. The general character of the components is the same. The chief reason for thinking that there may be a real difference between electrical components under different conditions is the fact that Stark[1] found no effect in the H and K lines of calcium (transverse). It was doubtful at first whether the production of spectra through the bombardment of metallic electrodes was favorable to an electrical effect. The fact that using metallic calcium as an electrode gave the same result as covering the electrode with calcium chloride proves this to be an entirely favorable method. The photographs with metallic calcium were unsatisfactory, owing to the tendency of calcium to act as an anode, but a few good ones obtained in this way showed the electric effect quite plainly.

The result most clearly shown by this work is the absolute necessity of increasing the intensity of the source so that high dispersion may be used. The effect in the majority of elements is so small that it will take high dispersion to bring it out.

GENERAL CONCLUSIONS

The spectra of H, Li, Ca, Fe, Ni, Mg, Al, Zn, and Sr were examined near the cathode in a discharge tube under low dispersion. Fe, Ni, Mg, Al, Zn, and Sr showed no electric effect. This

[1] *Annalen der Physik*, **43**, 1017, 1914.

negative result brings out the absolute necessity of increasing the dispersion in dealing with the heavier elements. The effect, if it exists in these elements, is small and can be brought out only under much stronger fields and with higher dispersion. Both Stark's method and that of Lo Surdo give such faint luminosity that the present problem is to find a method for producing very brilliant spectra in a strong electric field.

New results were found in lithium and calcium which show most interesting variations from those previously obtained.

The longitudinal effect in lithium gives clearly polarized components, whereas all the longitudinal components in hydrogen and helium have been unpolarized.

The H and K lines of calcium show electrical components with a separation, especially in the longitudinal effect, which is comparable in magnitude with the separations in H, He, and Li. This is surprising, owing to the relatively large atomic weight of calcium and to the fact that H and K do not belong to a diffuse series.

In conclusion, I wish to express to Dr. Hale and the members of the Observatory staff my very deep appreciation for the opportunity of a year's work at the Observatory. The problem of the electric effect was undertaken at Dr. Hale's suggestion, and has been guided by his advice and encouragement throughout.

Mount Wilson Solar Observatory
July 1, 1916

SOME DETERMINATIONS OF THE APEX AND VELOCITY OF SOLAR MOTION FROM THE RADIAL VELOCITIES OF THE BRIGHTER STARS, INCLUDING AN APPARENT RELATION TO PROPER MOTION

By C. D. PERRINE

In a paper[1] on the residual radial velocities of the stars of the different spectral classes contained in Campbell's well-known catalogues of about 1300 of the brighter stars, I pointed out several peculiarities, among which were large and apparently systematic residuals in the regions at right angles to the axis of solar motion.

Those investigations were based upon Campbell's assumed apex of $A=270^\circ$, $D=+30^\circ$. This position is only 5° away from that found by him from the radial velocities of these same stars and is midway between the latter value and the value derived by Boss from the proper motions of a larger number of (chiefly) naked-eye stars. Positions of the solar apex differing but little from that assumed by Campbell have been used by other investigators of solar motions. At that time it seemed unlikely that the most logical apex could differ sufficiently from the one chosen to give any very large discordances due to that cause alone, and attempts were made to find explanations in systematic motions of the stars themselves. It was found that the large systematic residuals in many of the galactic groups approximately at right angles to the axis of the solar motion could be largely accounted for by the assumption of a component of motion for the stars toward the vertex of one of Kapteyn's streams at 7^h, $+64^\circ$.

In attempting to find the cause of the apparently systematic discordance of nearly 10° between the declinations of the solar apex as derived from radial velocities and proper motions, the first step was a determination of the apex from the proper motions of the stars of Class B. It was at once seen that there was a marked discordance between the mean proper motions of the groups in the

[1] *Astrophysical Journal*, **42**, 305, 1915.

northern and southern hemispheres. This discordance was so marked and so suggestive that solutions were made for the apex and velocity of solar motion from the radial velocities of the northern and southern B stars separately. The results of these solutions and similar ones for some other classes yielded widely different positions of the solar apex.

At this point I received a letter from my friend, Professor Kapteyn, to whom I had written regarding the peculiarities found, in which he suggested that a different apex for the solar motion would remove the systematic residuals found in the regions of 0^h and 12^h of right ascension. This appears to be essentially the case. But I was now convinced that no single position of the apex would satisfy all of the discordances which I had found, not only from the radial velocities, but from the proper motions of the B stars as well. These discordances were believed to be something more than merely accidental. Separate solutions were made from the radial velocities of the north and south stars of all of the spectral classes. The separation into north and south groups divides the effect observed in the B stars on opposite sides of the axis of solar motion sufficiently at least for a preliminary investigation.

These solutions were first made for all of the stars of $3^M.0$ and fainter of each spectral class in each hemisphere. Such marked peculiarities were observed in the radial velocities of K-type stars within the same region, which had been separated according to size of proper-motion, and particularly in stars having contrary parallactic signs (i.e., negative signs in the first and fourth quadrants of right ascension and positive signs in the second and third quadrants), that ultimately the four classes A, F, G, and K were separated into four groups according to proper motion, viz., 0 to $0''.0049$, $0''.0050$ to $0''.0099$, $0''.0100$ and over, and those stars having "contrary" parallactic signs. The B stars practically all fell into the small proper motions, and the M-type stars and the 110 stars brighter than $3^M.0$ were not so classified, as they were thought to be too few in number.

The solutions were made by regions, each region being 2^h in right ascension by $30°$ in declination, commencing at 9^h and at the equator. The mean α, δ, and observed radial velocity of all of the

stars in each region were treated as one observation. The positions were rounded off to the nearest degree and the velocities to the nearest kilometer. In cases of but few stars observations were combined when they fell within an area differing but little from the assumed area, or rejected. Each region was given weight unity irrespective of the number of stars it contained or other conditions whose bearing for the present is unknown.

The equations of condition were of the well-known form

$$ax+by+cz+K-\text{Obs. }V=0$$

in which

$a=\sin\delta$,
$b=\cos\alpha\cos\delta$,
$c=\sin\alpha\cos\delta$,
$K=$ "constant error" term,
Obs. $V=$ observed radial velocity of star,
α and δ are the right ascension and declination of the star.

The solutions were made by the method of least squares. The A and D of the apex and the velocity $V_\odot$ were derived from the co-ordinates x, y, and z by the usual trigonometric relations, viz.,

$$V_\odot^2=x^2+y^2+z^2,$$

$$\tan A=\frac{z}{y},$$

$$\sin D=\frac{x}{V_\odot}.$$

In solutions of this nature, from small numbers of such different velocities as usually constitute each mean, a perfect representation of the spherical relations that exist among the coefficients is only accidental, even if there is no large systematic deviation. There is, therefore, generally an excess, sometimes considerable. This excess can be absorbed by the introduction of a constant-error term or other form of term which is independent of the spherical relations existing among the coefficients of the three unknown terms for determining the apex.

The term K, included largely for convenience in checking the solutions, was not used after that point. The values of the three unknowns x, y, and z are therefore independent of this excess (K).

If the K term were carried into the other unknowns, the values of y and z would have been considerably changed in only a few cases, but owing to the unsymmetrical nature of the coefficients in declination this co-ordinate of the solar apex is greatly changed by including a large K term. The evidently spurious values of such a term as resulted in many cases and their consequent effects on the declination of the apex caused me to discard entirely the effect of such a term in obtaining the position of the apex and the velocity.

In the earlier stages of the work the value of K was derived in the usual way from the solutions. It was often quite large and generally positive. The divisors for that term were, however, very small, owing to the unsymmetrical nature of the data, and it was evident that the effect of even one large discordant velocity was sufficient to yield an impossible value for K. It seems, therefore, that if we are to consider this as a constant error, we should, in place of the value derived for K (in these solutions at least), use the excess divided by the number of observations (regions in this instance). Values of K derived in this way are included in Table I with other data.

The stars with velocities of 50 km and over were generally omitted. The different classes of proper motion are indicated by large, medium, small, and contrary.

Several peculiarities of an apparently systematic nature are noticeable in these results, which are shown more clearly by the groupings in Tables II, III, IV, V, and VI. They may be summarized as follows:

A. There are such differences between the results from the northern and southern stars as to lead to the belief that there are radical structural differences in the two regions.

There appears to be greater consistency generally among the northern stars. All of the apices derived from the northern stars are north of the galactic plane, whereas five of the apices from the southern stars are south of it. Those from the southern stars are more widely scattered and show preferences for regions of sky somewhat different from that preferred by the apices derived from northern stars. The five groups with southern galactic latitudes all belong to the large and medium proper motions.

TABLE I

Spectral Type	$\mu\alpha$	North						South					
		A	D	$V_\odot$	K	No. Stars	No. Equations	A	D	$V_\odot$	K	No. Stars	No. Equations
				km	km					km	km		
$2^{\mathrm{M}}.9$ and brighter	All	251°	+44°	−14.4	−1.2	50	17	258°	+40°	−24.5	+0.5	60	16
B $3^{\mathrm{M}}.0$ and fainter	All	254	+27	−17.0	+0.8	70	13	288	+31	−24.8	+0.6	123	18
A $3^{\mathrm{M}}.0$ and fainter	Large	242	+ 7	−14.2	+0.4	19	8	306	+14	−20.7	−0.1	10	7
	Medium	260	− 4	−22.6	0.0	19	8	305	+20	−14.5	−2.5	15	8
	Small	260	+35	−17.7	−0.5	59	17	261	+ 3	−20.2	+1.7	63	18
	Contrary	209	+37	−12.2	+0.6	32	13	240	+18	−18.9	+3.3	19	10
	All	258	+20	−17.7	+0.1	97	21	264	+11	−19.2	+0.8	88	18
F $3^{\mathrm{M}}.0$ and fainter	Large	254	+ 5	−23.2	−0.1	40	10	238	−11	−13.1	−2.1	38	14
	Medium	253	−17	−25.5	+0.3	12	6	307	+19	−19.9	−3.2	21	9
	Small	270	+37	−15.0	0.0	27	10	264	+32	−20.7	+0.8	46	15
	Contrary	292	+58	−13.3	−0.2	29	11	204	+32	− 8.4	−0.2	39	12
	All	271	+ 9	−19.2	+0.2	79	20	264	+13	−18.7	−0.1	105	21
G $3^{\mathrm{M}}.0$ and fainter	Large	268	− 4	−36.4	+4.0	19	11	287	− 6	−13.9	−7.5	17	10
	Medium	272	+ 7	−37.6	+2.6	9	8	246	− 6	−44.1	−0.3	5	5
	Small	271	+27	−14.6	−0.1	36	13	242	+28	−12.4	+0.3	45	17
	Contrary	207	+80	−19.0	+2.0	23	11	147	+ 1	− 6.4	−0.6	18	10
	All	267	+19	−20.9	+0.7	64	19	249	+23	−12.3	−1.0	67	19
K $3^{\mathrm{M}}.0$ and fainter	Large	277	+18	−26.3	−0.2	34	14	298	+19	−30.0	+1.8	51	17
	Medium	262	−34	−25.2	+3.0	34	14	232	+59	−21.3	−1.2	51	19
	Small	265	+32	−19.6	+0.5	82	20	258	+39	−22.4	+1.2	138	25
	Contrary	288	+44	−14.2	−0.3	39	14	292	+60	−20.3	+0.7	57	17
	All	273	+ 8	−19.4	+2.3	151	28	268	+36	−22.2	+0.8	240	35
M $3^{\mathrm{M}}.0$ and fainter	All	269	+15	−24.1	+1.1	24	11	272	+41	−22.5	+2.2	41	20

TABLE II

ACCORDING TO PROPER MOTION

μα	Spectral Type	North						South					
		A	D	$V_\odot$	K	No. Stars	No. Equations	A	D	$V_\odot$	K	No. Stars	No. Equations
				km	km					km	km		
Large	A	242°	+ 7°	−14.2	+0.4	19	8	306°	+14°	−20.7	−0.1	10	7
	F	254	+ 5	−23.2	−0.1	40	10	238	−11	−13.1	−2.1	38	14
	G	268	− 4	−36.4	+4.0	19	11	287	− 6	−13.9	−7.5	17	10
	K	277	+18	−26.3	−0.2	34	14	298	+19	−30.0	+1.8	51	17
		260	+ 6	−25.0	+1.0	112		282	+ 4	−19.4	−2.0	116	
Medium	A	260	− 4	−22.6	0.0	19	8	305	+20	−14.5	−2.5	15	8
	F	253	−17	−25.5	+0.3	12	6	307	+19	−19.9	−3.2	21	9
	G	272	+ 7	−37.6	+2.6	9	8	246	− 6	−44.1	−0.3	5	5
	K	262	−34	−25.2	+3.0	34	14	232	+59	−21.3	−1.2	51	19
		262	−12	−27.7	+1.5	74		272	+23	−25.0	−1.8	92	
Small	A	260	+35	−17.7	−0.5	59	17	261	+ 3	−20.2	+1.7	63	18
	F	270	+37	−15.0	0.0	27	10	264	+32	−20.7	+0.8	46	15
	G	271	+27	−14.6	−0.1	36	13	242	+28	−12.4	+0.3	45	17
	K	265	+32	−19.6	+0.5	82	20	258	+39	−22.4	+1.2	138	25
		266	+33	−16.7	0.0	204		256	+26	−18.9	+1.0	292	
Contrary	A	209	+37	−12.2	+0.6	32	13	240	+18	−18.9	+3.3	19	10
	F	292	+58	−13.3	−0.2	29	11	204	+32	− 8.4	−0.2	39	12
	G	207	+80	−19.0	+2.0	23	11	147	+ 1	− 6.4	−0.6	18	10
	K	288	+44	−14.2	−0.3	39	14	292	+60	−20.3	+0.7	57	17
		249	+55	−14.7	+0.5	123		221	+28	−13.5	+0.8	133	
	Omitting G	263	+46	−13.2				245	+37	−15.9			

B. The position of the solar apex varies with the size and parallactic sign of the proper motions of the stars used in the determination, being for the northern sky progressive in *declination* from the large and medium values through the small values to those with

TABLE III

According to Declination of Apex

North					South				
Spectral Type	μ_α	A	D	$V_\odot$	Spectral Type	μ_α	A	D	$V_\odot$
				km					km
K	M	262°	−34°	−25.2	F	L	238°	−11°	−13.1
F	M	253	−17	−25.5	G	L	287	− 6	−13.9
A	M	260	− 4	−22.6	G	M	246	− 6	−44.1
G	L	268	− 4	−36.4	A	S	261	+ 3	−20.2
		261	−15	−27.4			258	− 5	−22.8
F	L	254	+ 5	−23.2	A	L	306	+14	−20.7
A	L	242	+ 7	−14.2	A	Con.	240	+18	−18.9
G	M	272	+ 7	−37.6	F	M	307	+19	−19.9
M	All	269	+15	−24.1	K	L	298	+19	−30.0
K	L	277	+18	−26.3	A	M	305	+20	−14.5
		263	+10	−25.1			291	+18	−20.8
B	All S	254	+27	−17.0	G	S	242	+28	−12.4
G	S	271	+27	−14.6	B	S	288	+31	−24.8
K	S	265	+32	−19.6	F	S	264	+32	−20.7
A	S	260	+35	−17.7	F	Con.	204	+32	− 8.4
		262	+30	−17.2			250	+31	−16.6
F	S	270	+37	−15.0	K	S	258	+39	−22.4
A	Con.	209	+37	−12.2	$2^{\mathrm{M}}9$ and brighter	All	258	+40	−24.5
K	Con.	288	+44	−14.2	M	All	272	+41	−22.5
F	Con.	292	+58	−13.3	K	M	232	+59	−21.3
$2^{\mathrm{M}}9$ and brighter		251	+44	−14.4	K	Con.	292	+60	−20.3
		262	+44	−13.8			262	+48	−22.2

contrary parallactic signs. The progression for the northern stars is very consistent, not a single exception being observed to the condition that the large and medium proper-motion stars yield positions *south* and the apices from the contrary proper-motion stars *north* of that from the small proper-motion stars.

The most southerly positions of the apex appear to result from the medium-sized proper motions in the classes A, F, and K.

TABLE IV

ARRANGED ACCORDING TO A

North				South			
Type	A	D	$V_\odot$	Type	A	D	$V_\odot$
			km				km
G.........	207°	+80°	−19.0	G.........	147°	+ 1°	− 6.4
A.........	209	+37	−12.2	F.........	204	+32	− 8.4
	208	+58	−15.6		176	+16	− 7.4
A.........	242	+ 7	−14.2	K.........	232	+59	−21.3
				F.........	238	−11	−13.1
$2^{\mathrm{M}}9$ and brighter..	251	+44	−14.4	A.........	240	+18	−18.9
F.........	253	−17	−25.5	G.........	242	+28	−12.4
F.........	254	+ 5	−23.2	G.........	246	− 6	(−44.1)
B.........	254	+27	−17.0				
	251	+13	−18.9		240	+18	−16.4
A.........	260	− 4	−22.6				
A.........	260	+35	−17.7	$2^{\mathrm{M}}9$ and brighter..	258	+40	−24.5
K.........	262	−34	−25.2	K.........	258	+39	−22.4
K.........	265	+32	−19.6	A.........	261	+ 3	−20.2
M.........	269	+15	−24.1	F.........	264	+32	−20.7
G.........	268	− 4	−36.4				
					260	+28	−22.0
	264	+ 7	−24.3				
F.........	270	+37	−15.0	M.........	272	+41	−22.5
G.........	271	+27	−14.6				
G.........	272	+ 7	−37.6				
K.........	277	+18	−26.3				
	272	+22	−23.4				
K.........	288	+44	−14.2	G.........	287	− 6	−13.9
F.........	292	+58	−13.3	B.........	288	+31	−24.8
				K.........	292	+60	−20.3
	290	+51	−13.8	K.........	298	+19	−30.0
				A.........	305	+20	−14.5
				A.........	306	+14	−20.7
				F.........	307	+19	−19.9
					298	+22	−20.6

There is some reason to think that this is a real condition, especially when we consider the consistency within the K-type groups which contain the largest number of stars and the fact that, after omitting

a part of the stars (about the ellipsoidal vertex), we still find the same strongly marked effect.

In the G-type stars the most southerly declination for the apex results from the largest proper motions. In many ways the motions of these stars, which are so closely related to our sun in spectral class, appear to be different from all of the other spectral classes. One explanation of these peculiarities which suggests itself is that the general motion of the G-type stars may approximate more nearly to that of our sun than the other types.

TABLE V

Apex from Small Proper Motions

Type	North				South			
	A	D	$V_\odot$	No. Stars	A	D	$V_\odot$	No. Stars
			km				km	
B............	254°	+27°	−17.0	70	288°	+31°	−24.8	123
A............	260	+35	−17.7	59	261	+ 3	−20.2	63
F............	270	+37	−15.0	27	264	+32	−20.7	46
G............	271	+27	−14.6	36	242	+28	−12.4	45
K............	265	+32	−19.6	82	258	+39	−22.4	138
	264	+32	−16.8	274	263	+27	−20.1	415
Omitting A.....	....	....	....	....	....	+32	....	....

For the southern stars this progression in declination is less marked, and there is also a progression in the right ascensions of the apex, decreasing from the large proper motions, through those of medium and small size, to those with contrary parallactic signs.

The positions of the apex from the four groups each of northern and southern stars given in Table II fall reasonably well on great circles, which make considerable angles with the galactic plane and with each other.

It should be remembered in this connection that to some extent the largest and smallest proper motions prefer essentially different regions of sky. The evidence from the stars of Class B, however, which have small proper motions in all regions where they are found, indicates that the variation of declination depends at least in part upon the size of the proper motions. While it seems not improbable that any dependence upon size of proper motion may be a

TABLE VI

EFFECT OF OMITTING STARS NEAR ELLIPSOIDAL VERTICES

STARS OF $3^{M}.0$ AND FAINTER	North						South					
	A	D	$V_{\odot}$	K	No. Stars	No. Equations	A	D	$V_{\odot}$	K	No. Stars	No. Equations
			km	km					km	km		
B	253°9	+26°5	−17.0	+0.8	70	13	287°7	+30°8	−24.8	+0.6	123	18
Omitting ellipsoidal vertices	253.5	+28.3	−17.5	+0.4	45	10	284.3	+49.4	−26.4	−3.6	84	15
A	256.9	+23.9	−19.6	+0.9	96	15	262.4	+13.5	−20.2	+0.7	85	16
Omitting ellipsoidal vertices	256.8	+34.8	−16.0	0.0	72	11	263.5	+11.6	−21.6	+1.2	71	12
F and G galactic	254.8	+22.0	−21.5	+0.2	66	12	274.2	+13.8	−12.0	−0.6	88	14
Omitting ellipsoidal vertices	257.5	+23.9	−21.1	+0.5	45	8	218.9	+24.3	−50.0	+1.4	53	9
K . . . Large	276.9	+17.8	−26.3	−0.2	34	14	297.8	+18.8	−30.0	+1.8	51	17
Omitting ellipsoidal vertices	288.5	+ 7.8	−28.1	−0.7	30	11	292.1	+ 6.8	−49.6	−0.1	40	13
Medium	262.2	−34.0	−25.2	+3.0	34	14	232.0	+58.6	−21.3	−1.2	51	19
Omitting ellipsoidal vertices	237.3	−42.4	−16.8	+1.2	25	11	220.0	+57.3	−23.3	−1.4	38	14
Small	264.7	+31.6	−19.6	+0.5	82	20	258.3	+39.4	−22.4	+1.2	138	25
Omitting ellipsoidal vertices	250.2	+43.8	−19.9	0.0	65	16	280.0	+18.2	−24.7	+0.7	98	18
Contrary	287.7	+44.1	−14.2	−0.3	39	14	292.2	+60.4	−20.3	+0.7	57	17
Omitting ellipsoidal vertices	Apparently no effect						305.5	+49.2	−20.2	+0.7	42	12
M	269.2	+15.0	−24.1	+1.1	24	11	272.2	+41.4	−22.5	+2.2	41	20
Omitting ellipsoidal vertices	275.0	+33.3	−19.0	+0.3	20	9	278.9	+ 7.8	−37.1	+1.5	26	13

dependence upon distance or other conditions, it seems almost necessary to conclude that the underlying cause is some form of rotary or spiral motion.

C. The solar velocity also varies in the northern sky with the size of the proper motions and the declination of the apex—the velocity decreasing from 27 km for the mean declination of apex $-15°$ to 14 km for the mean declination of apex $+44°$. This progression is fairly consistent, as shown in Table III. There is some indication that in the southern stars the *smaller* solar velocities go with the more southerly apices.

D. The solar velocity appears to vary in both hemispheres with the galactic latitude of the apex.

E. The declinations of the apex from the small proper motions of five of the spectral classes are with one exception quite accordant from both northern and southern stars, and do not differ greatly in the mean from that generally used by investigators of stellar motions. The conclusion is reached that the small declinations obtained for the solar apex from radial velocities have been due very largely to the influence of stars of medium and large proper motions.

F. With the exception of the stars of Class G, the velocity of the solar motion from the small proper-motion stars is consistently less for the northern than for the southern stars. Slightly the reverse appears to be true for the other classes of proper motions, but this conclusion is not very reliable.

If the stars about the ellipsoidal vertices are omitted, the solar velocities in the northern sky are, without exception in the eight general divisions of Table VI, smaller than in the southern sky.

G. The solar velocities from small proper-motion stars are less from the middle-type stars than from the early and late types. This may possibly be due to a larger admixture of stars with contrary parallactic components of motion.

H. The value of the solar velocity increases generally from the stars with contrary parallactic signs to those with large proper motions. Some such result was to be expected from a more or less accidental distribution of motions and distances.

I. The values of the constant error as found above from northern and southern stars separately are generally small and not very different in the different spectral types.

J. The systematic variation in the declination of the apex from the different spectral classes, which is shown in Table VII, appears to be largely due to the proportion of stars with large and medium-sized proper motions which they contain.

TABLE VII

FROM ALL REGIONS OF SKY

Spectral Type	A	D	$V_\odot$	K	No. Stars	No. Equations
			km	km		
$2^{M}.9$ and brighter.........	258°.0	+41°.5	−18.9	+1.0	110	33
B $3^{M}.0$ and fainter	276.0	+29.6	−20.3	+4.0	193	31
A $3^{M}.0$ and fainter	260.9	+15.3	−18.3	−0.1	185	39
F $3^{M}.0$ and fainter	267.9	+11.1	−18.9	+0.4	184	41
G $3^{M}.0$ and fainter	257.4	+20.2	−16.4	−0.7	131	38
K $3^{M}.0$ and fainter	274.2	+25.6	−20.7	+4.3	391	63
M $3^{M}.0$ and fainter	269.7	+31.7	−22.9	+4.0	65	31

K. Marked excesses of positive velocities have been found in the K- and M-type stars in the regions of the ellipsoidal vertices. In some groups these excesses are as large as the solar velocity itself. There appears to be little or no such effect in the early and middle classes. Table VI contains the results of solutions including and omitting the groups within about 40° of the ellipsoidal vertices. Owing to the unsymmetrical nature of the data in the classes limited to galactic stars, the results omitting the vertices are rather uncertain. This is particularly true of the southern F and G stars. At the present time it does not seem necessary to try to strengthen those particular results.

These large excesses of positive velocities in the regions of the ellipsoidal vertices appear to be arranged in streams rather than symmetrically about an axis.

L. There are some preferences in the right ascensions of the apex, which if confirmed are undoubtedly significant. Those from the B stars may be taken as probably the most trustworthy. The right ascension of the apex from northern B stars differs over $30°$ from that derived from the southern stars, a difference which is not greatly changed by the omission of the stars about the ellipsoidal vertices, one-third of the total number. Something similar was found in the right ascensions of the apex of these stars derived from their proper motions.

It is perhaps suggestive that in the considerable region along which these positions of the apex are distributed we find the Milky Way divided into two well-defined streams of the same order of distance apart as these apices.

The progressive change in the position of the solar apex *toward the ellipsoidal axis* as we go from the stars of small to those of large proper motion is very significant.

The fact that any particular class of stars (in this case large and medium-sized proper motion) gives positions of the solar apex not far from the ellipsoidal axis would seem definitely to connect the phenomena observed with those of star-streaming as explained by the ellipsoidal hypothesis. The larger solar velocities from the northern stars giving such declinations for the apex appear also to confirm a close relation with ellipsoidal phenomena.

Discussion of these results is reserved until investigations have been made of other conditions which should also bear upon an explanation, such as proper motions, distances, the distributions of residual velocities and of the relations which appear to exist among the different conclusions themselves. The general conclusion may be stated, however, that the apparently systematic differences in the position of the apex and in the velocity of the solar motion strongly indicate, if they do not establish, variations of the general direction and velocity of motion of the stars themselves in different portions of our stellar system.

Systematic variation of the position of the apex of solar motion seems to be as definitely indicated as other conclusions which have been drawn from these same stars (1300 in number), such as increase

of velocity with increasing supposed age of the stars, constant-error term, magnitude-velocity equation, etc. If some such variation is confirmed, then we are confronted with the necessity of choosing some position of the apex as fundamental. Several bases for such a decision suggest themselves now, such as faint or very faint stars; small proper motions; large proper motions; the more distant stars; the nearer stars; stars which after a better knowledge of their motions have been obtained are known to be themselves nearly stationary; classes of stars having peculiar qualities (now unknown perhaps) which better fit them for this purpose than the condition mentioned. In any case a large number of objects is of course essential. If great difficulty is encountered in finding a suitable basis among the objects in our own stellar system, it might even be necessary or desirable to use the objects of external systems, as, for example, the spiral nebulae.

It would seem, however, that a better knowledge of the peculiarities of motions in our stellar system, together with a study of the structure of the Milky Way, should furnish a satisfactory basis for the choice of a fundamental apex.

The results of general solutions for all parts of sky for each spectral class separately are given in Table VII.

Observatorio Nacional Argentino, Córdoba
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A PHOTOMETRIC STUDY OF THE ECLIPSING VARIABLE RZ CASSIOPEIAE

By R. S. DUGAN

The variability of RZ Cassiopeiae was discovered by Müller in 1906. He found it to be of the eclipsing type. The favorable position and brightness of the star, the short period of revolution and duration of eclipse made it an attractive object for observers. It has been quite extensively observed visually with and without photometers, and photographically in focus and out of focus, with and without color-screens.[1] RZ Cassiopeiae is B.D. $+69°179$, and its provisional designation as a variable was 77.1906. It is of type A.

Certain questions were, however, left unanswered by this mass of material. I started observing this star in 1911 with the hope of deciding whether we had at last an eclipsing variable without measurable secondary minimum, and to measure the minor effects of ellipticity and interchange of radiation. The 9792 measures necessary to answer these questions satisfactorily were made with the polarizing photometer in the usual manner. Corrections for atmospheric absorption have been applied. Two comparison stars were used. B.D. $+69°184 = a$, which was used in the early measures, proved to be variable through a small range. The later observations with B.D. $+69°181 = b$ are sufficiently numerous to

[1] Müller and Münch, *Astronomische Nachrichten*, **171**, 357, 1906; **183**, 76, 1909; Nijland, *ibid.*, **176**, 171, 1907; Lehnert, *ibid.*, **192**, 201, 1912; **194**, 165, 1913; Hoffmeister, *ibid.*, **202**, 42, 1916; **197**, 317, 1914; Parkhurst and Jordan, *Astrophysical Journal*, **26**, 251, 1902; Wendell, *Harvard Annals*, **69**, Part 2; Graff, *Mitteilungen der Hamburger Sternwarte*, No. 13; Beljawsky, *Mitteilungen der Nikolai-Hauptsternwarte zu Pulkow*, **3**, 31, 1910; Bemporad, *Memorie della Società degli Spettroscopisti italiani* (2), **2**, 153, 1913; Padova, *ibid.* (2), **2**, 57, 1913; Lazzarino, *ibid.* (2), **2**, 123, 1913; Jordan, *Publications of the Allegheny Observatory*, **3**, No. 16; Harvard photographic observations, received in manuscript; Yendell, *Astronomical Journal*, **28**, 126, 1914; Venturi Lacchini, *Riv. Astr.*, **7**, 241, 1913.

define the light-curve of RZ Cassiopeiae and hence the variations of star *a*.[1]

A careful study of all observations of RZ Cassiopeiae during eclipse, including the Harvard photographs, resulted in the clear detection of a variation of the period. A paper dealing with this matter is soon to appear in the *Monthly Notices*.

The observations were combined into normals, including, on the average, five sets of sixteen measures. The $a-v$ observations are reduced to $b-v$ by adding $0^{M}.415$.

As the depth of secondary minimum (d) is comparable to the effects of ellipticity (c) and exchange of radiation or reflection (b), a least-square solution was made including all normals not observed during primary minimum. The light (a) of the system at longitude 90°, and the time of secondary mid-eclipse were two further unknowns.[2] The solution resulted in the following set of values:

$$a=0.961\pm0.0034$$
$$b=0.029\pm0.0038$$
$$c=0.016\pm0.0069$$
$$d=0.058\pm0.0068$$

$$\text{Time of sec.}=\text{time of primary}+\tfrac{1}{2}P-4^{m}.4\pm5^{m}.6.$$

The unit of intensity is $b-v=3^{M}.020$.

The shift of secondary minimum, found from the solution, was disregarded, because it evidently results from two low normals in the early part of secondary minimum. It is furthermore not at all comparable with the shift to be expected from the position of periastron and the eccentricity found from the spectrographic observations at Allegheny, and is in the opposite sense to the shift required by the interpretation of the change in the period as due to a revolution of the line of apsides. This assumption of a circular orbit makes no appreciable difference in the results.

The elements of the system RZ Cassiopeiae derived from the mean light-curve are shown in Table I.

The unit of light is the combined light of the brighter sides of the two stars at their maximum area, corresponding to $b-v=3^{M}.016$. The unit of length is the mean radius of the relative orbit.

[1] *Astronomical Journal*, **29**, 137, 1916.

[2] Russell, *Astrophysical Journal*, **39**, 407, 1914.

In combination with the spectrographic results[1] we obtain the following absolute elements, given in Table II.

TABLE I

	Uniform	Darkened
Maximum radius of larger star, a_f	0.2966	0.2886
Minimum radius of larger star, b_f	0.2917	0.2857
Maximum radius of smaller star, a_b	0.2278	0.2759
Minimum radius of smaller star, b_b	0.2240	0.2730
Ratio of the radii of the stars, k	0.768	0.956
Ratio of the axes of the spheroidal stars, $1+\frac{1}{2}z$	1.016	1.010
Least apparent distance of centers, cos i	0.153	0.147
Inclination of orbit plane, i	81°10′	81°34′
Eccentricity of orbit, e	0	0
Maximum fraction of light of smaller star obscured during primary eclipse, α_0	0.806	0.770
*Difference of light of sides of larger star, $2b_1$	0.063	0.063
Difference of light of sides of smaller star, $2b_2$	0.006	0.006
Light of brighter side of smaller star, L_b	0.877	0.918
Light of brighter side of larger star, L_f	0.123	0.082
Ratio of surface brightness:		
Of the bright sides of the two stars, J_b/J_f	12.1	12.3
Of the sides of the fainter star	1.51	1.77

* The quantity b in the least-squares solution is the difference of the reflection effects b_1-b_2 on the two stars. This separation assumes that the quantities b are proportional to the energy received by the surface of each star from the radiation of its companion.

TABLE II

	$m_b=2\,m_f$		$m_b=2.7\,m_f$	
	Uniform	Darkened	Uniform	Darkened
Max. radius larger star, in km, a_f	1,024,000	996,000	1,263,000	1,229,000
Max. radius smaller star, in km, a_b	786,000	953,000	970,000	1,175,000
Mass of larger star, m_f	0.38 ☉		0.58 ☉	
Mass of smaller star, m_b	0.77 ☉		1.58 ☉	
Density of larger star, ρ_f	0.13 ☉	0.14 ☉	0.10 ☉	0.11 ☉
Density of smaller star, ρ_b	0.56 ☉	0.31 ☉	0.61 ☉	0.34 ☉
Radius of orbit, in km	3,452,000		4,258,000	

The second assumption of the mass-ratio is based on the conclusions reached by Shapley in his study of relative mass and relative brightness in spectroscopic and visual binaries.[2] Some

[1] *Publications of the Allegheny Observatory*, 3, No. 16.

[2] *Contributions from the Princeton University Observatory*, No. 3, p. 121.

assumption of the mass-ratio is necessitated by the faintness of the large star and the consequent failure to observe the lines of its spectrum with the spectrograph. The two assumptions made above are regarded as reasonable on several grounds.

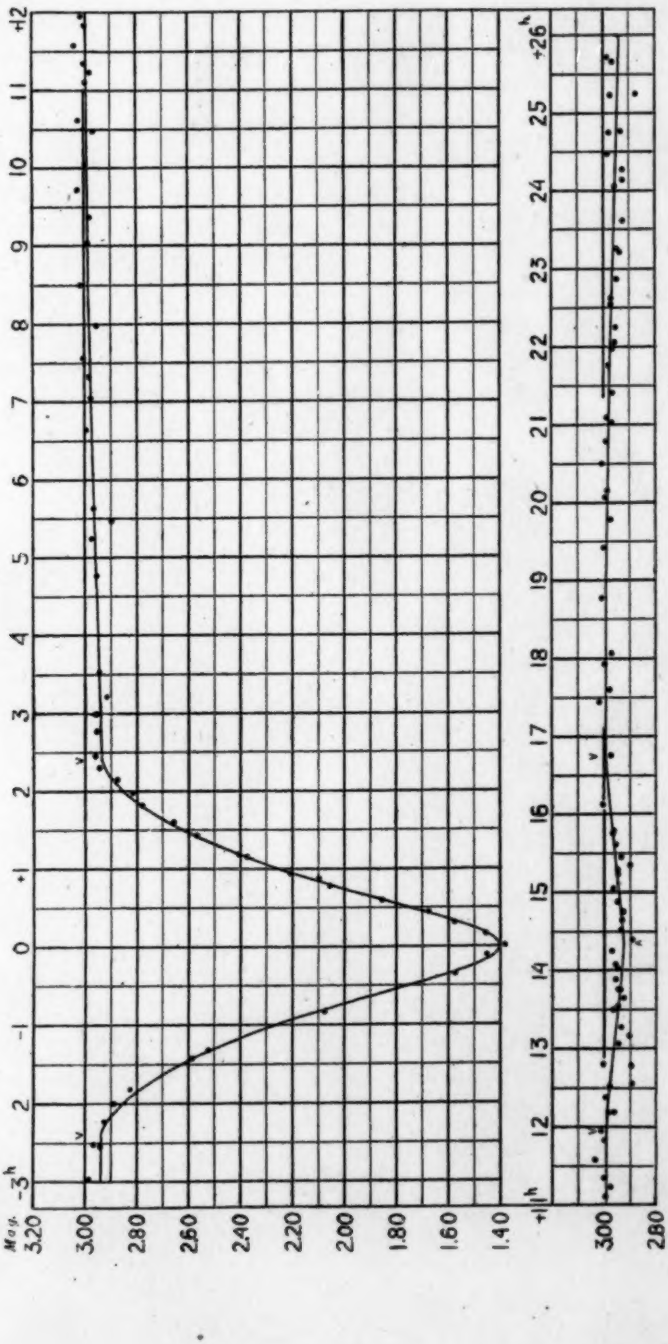
The residuals of the darkened solution are identical with those of the uniform solution with the exception of the regions $-2^h38^m.7$ to -1^h30^m and $+1^h30^m$ to $+2^h38^m.7$. In these regions the darkened curve lies below the uniform, the maximum difference being $0^M.026$. The uniform curve is apparently the better fit.

The probable error of a single set of sixteen measures, from the residuals of the uniform solution, is $\pm 0^M.037$. The corresponding figures for stars previously investigated by the writer were: RT Persei, $\pm 0^M.037$; Z Draconis, $\pm 0^M.039$; RV Ophiuchi, $\pm 0^M.051$. The last-mentioned star shows a decidedly asymmetrical light-curve, with the resulting large probable error.[1]

The Harvard photographic observations sent me, from which I formed a mean curve for the study of the period, are hardly numerous enough to warrant an independent solution. The photographic range of primary minimum is about $0^M.23$ greater than the visual range.

A comparison of Wendell's observations with my own should prove interesting, since they were made with the same kind of photometer. Wendell used star *b* for comparison, and made 312 sets of sixteen measures. Nearly 200 of the 312 were made during the five hours of primary minimum, leaving the other twenty-four hours of the curve not very well defined by the observations. No effects of ellipticity or reflection are apparent. There is some indication of a secondary minimum of about the depth shown by my curve, but displaced nearly two hours toward the following primary. The observations are not sufficiently numerous to determine these three quantities with much accuracy. The mean epoch of Wendell's observations of secondary minimum is about 900 periods earlier than mine. The brightness at primary minimum is practically identical in the two curves. The difference is at most one one-hundredth of a magnitude. The uniform curve computed from my observations follows the observations of Wendell perfectly

[1] *Astrophysical Journal*, **43**, 130, 1916.



Computed Uniform Mean Light-Curve of RZ Cassiopeiae

from the bottom of primary minimum as far as phase $\pm 1^h 20^m$. From there on the observations of Wendell drop steadily more and more below my curve, the difference at the end of primary eclipse being $0^M.04$. The average of Wendell's observations outside primary eclipse is $b-v=2^M.90$, while the average of mine is $b-v=2^M.966$. As the difference in the two curves is probably due to different color-sensitiveness of the two observers and the two telescopes, it appears probable that the color of the variable approaches that of the comparison star at mid-eclipse. An independent solution of Wendell's primary minimum, using the ellipticity and reflection coefficients, and the depth of secondary minimum found from my observations, resulted in the following somewhat different set of uniform elements: $k=0.612$, $\alpha_0=0.844$, $i=79°28'$, $a_f=0.3105$, $a_b=0.1900$, $L_b=0.815$, $\frac{J_b}{J_f}=11.3$.

SUMMARY

1. The following new eclipse elements have been found for the system RZ Cassiopeiae:

J.D. $2417355.4208+1^d.19525\,E+0^d.007 \sin(12°+0°.068\,E)=1906$ May $24^d 10^h 6^m.0$ G.H.M.T.$+1^d 4^h 41^m\, 9^s.6\,E+10^m \sin(12°+0°.068\,E)$.

2. At primary minimum, when eight-tenths of the smaller brighter component is covered by the larger fainter component, the star is $1^M.59$ fainter than at maximum. The loss of light at secondary minimum is $0^M.06$. Every eclipsing variable which has been observed with sufficient care and persistence shows a measurable secondary minimum.

3. The ellipticity and reflection effects detected in the curve are comparable with those found in the study of other systems.

4. The smaller star emits 7 times as much light as the larger, while its surface brightness is 12 times as great. The distance between centers is $3\frac{1}{2}$ times the radius of the fainter star, and probably between 5 and 6 times the radius of the sun. The brighter component is probably between $2\frac{1}{2}$ and 6 times as dense as the fainter component.

5. A comparison of my own observations with the Harvard photographic and Wendell's photometric observations shows that the star is redder than the comparison star B.D.$+69^\circ 184$ and approximates it in color more closely at minimum than at maximum light.

Princeton University Observatory
May 13, 1916

Minor Contributions and Notes

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE

A determination of the least quantity of radiant energy capable of exciting the sensation of light could and probably by preference should be made in the laboratory by a direct method. The existence of a commonly accepted standard of just visibility, namely, the sixth-magnitude star, permits a determination of this quantity (under certain limiting conditions) by a somewhat indirect method. Drude[1] in his *Lehrbuch der Optik* calculates this quantity in this way as 0.6×10^{-8} ergs per second, assuming a pupillary diameter of 3 millimeters. Unfortunately Drude's treatment of this problem suffers from the errors incident to the crude and inaccurate manner of handling the radiation-light relations which was in vogue when he wrote. His result is in error, for the size of pupil assumed, by a factor of about 10. The object of this note is the recalculation of this least-perceptible quantity of radiation, using methods free from objection, and at the same time taking advantage of the latest data on the relation of stellar magnitudes to terrestrial-light standards.

The steps followed by Drude are as follows: He first states the "mechanical equivalent" of the Hefner unit of light, meaning by this the radiation lying between the "visible" limits of the spectrum corresponding to 1 (Hefner) lumen. For this he takes the experimental figure of Ångström, namely, 0.8×10^{-5} ergs per second. From this he calculates that 1 (Hefner) meter-candle is equivalent to 8.1 ergs per second per square centimeter. He then notes that a sixth-magnitude star gives an illumination of 10^{-8} meter-candles since it appears as bright as a Hefner lamp at 11 kilometers distance. Taking the pupillary opening as 3 millimeters, he arrives by simple multiplication at his figure of 0.6×10^{-8} ergs per second as the radiation entering the eye.

[1] Drude, *Lehrbuch der Optik*, 2d ed., p. 471.

The fundamental error in this procedure is the assumption that all "visible" radiation has the same value as light, that is, as measured on a photometer. Actually equal amounts of "visible" radiation from an approximately white star and from the very red Hefner lamp would measure several times different on a photometer, while if the radiations were all concentrated in the most efficient part of the spectrum for light-production, the amount of "visible" radiation given out by the Hefner would yield nine to ten times the light it does. What Drude wished to determine was the *least* amount of radiation visible as light, for which he should have taken as his standard the most efficient possible radiation from the standpoint of light-production, while the Hefner is about the least efficient light-source finding any use at the present time.

In passing, it may be pointed out that no more complete proof of the inadequacy of the old purely physical definitions of "luminous efficiency" and the "mechanical equivalent of light" could be found than in this same chapter of Drude, where on this fundamental assumption that all visible radiation has the same light-value he proceeds to calculate the "luminous efficiency" of the arc lamp from its candles per watt, and the illumination due to sunlight from its "luminous efficiency" and the solar constant. In the one case he arrives at a figure much higher than any arrived at by experiments based on the same criterion of luminous efficiency; in the other case, for the same reason, he comes out with much less than the value obtained by direct measurements. It is to be hoped that later editions of this otherwise admirable textbook will have this chapter recast.

Without going into details, for which the reader is referred to previous papers of the writer,[1] suffice it to say that the process gone through by Drude is legitimate and exact, provided the crude definition of luminous flux as radiation lying between certain spectral limits is superseded by the definition that it is radiation evaluated according to its capacity to produce the sensation of light, that is, according to the luminosity-curve of the spectrum.

[1] Ives, "The Primary Standard of Light," *Astrophysical Journal*, **36**, 322, 1912; Ives, "The Establishment of Photometry on a Physical Basis," *Journal of the Franklin Institute*, **180**, 409, 1915.

Radiation thus evaluated *is* directly proportional to the photometric value of the light produced. The factor of proportionality is called, using the old misapplied term, the "mechanical equivalent of light." Experimentally this has been determined as 0.00159 watt per lumen.[1] This mechanical equivalent of light is the *least* quantity of radiation which can produce one lumen of luminous flux.

We are now in position to make the calculation which is the object of this paper. We first note that

1 meter-candle = 1 lumen per square meter = 0.0001 lumen per square centimeter = 0.000000159 watt per square centimeter = 1.59 ergs per second per square centimeter.

(On the basis of a 3-millimeter diameter pupil the amount of radiation entering the eye *from the most efficient unit light-source* at 1 meter would raise 1 gram of water 1° Centigrade in something over eleven years.)

In order to find the illumination due to a sixth-magnitude star it is necessary to know the relationship between the stellar-magnitude scale and the candle-power scale. This has recently been discussed by Russell,[2] who gives as the weighted mean of several determinations in which the comparisons were made at color-match, that is, as though at high illuminations, that

1 candle at 1 meter is of stellar magnitude -14.18.

By the ordinary formula for reducing stellar magnitudes to intensities we find that the brightness of a sixth-magnitude star is

$$0.849\times10^{-8} \text{ of this;}$$

hence the least power corresponding to illumination from a light-source of this brightness is

$1.59\times0.849\times10^{-8}$ ergs per second per square centimeter $=1.35\times10^{-8}$ ergs per second per square centimeter.

Drude assumed the diameter of the pupil to be 3 millimeters. This is probably too low, as under the conditions of nocturnal observation it would be fully dilated, probably to a diameter of

[1] Ives, Coblentz, and Kingsbury, "The Mechanical Equivalent of Light," *Physical Review*, **5**, 269, 1915; Ives and Kingsbury, "Physical Photometry with a Thermopile Artificial Eye," *Physical Review*, **6**, 319, 1915.

[2] Russell, "The Stellar Magnitudes of Sun, Moon, and Planets," *Astrophysical Journal*, **41**, 103, 1916.

6 or 7 millimeters. Taking 6 millimeters as a reasonable estimate, it follows that the radiation entering the eye from a light-source of maximum efficiency of the brightness of a sixth-magnitude star would be

$$0.38\times10^{-8}\ \frac{\text{ergs}}{\text{sec. sq. cm}}.$$

This then is, on the assumptions made, the smallest amount of radiation perceivable by the eye. It is important to note, however, that this figure applies only to radiation from a distant-point source, e.g., a star. The energy is, of course, concentrated on the retina into an area of the size of the image formed, whereby the energy-density on the retina is greater than at the pupil by a factor of approximately 10^5. (A study of the visibility of large and small images of the same total intensity would be necessary in order to give the complete answer to the question under discussion.) The amount received by the retina is again reduced somewhat, owing to the absorption of the eye-media, which, however, are quite transparent for visible radiation.

No account has been taken in this calculation of the shift of the maximum of visual sensibility toward shorter wave-lengths at the low intensities of observation common in stargazing. This has been unnecessary because the connection between stellar and terrestrial magnitudes has been established, as stated, for high-illumination conditions, that is, for those by which the mechanical equivalent of light was determined. The only outstanding error then becomes the difference in area of the luminosity-curve of the spectrum of an observed star as the observing conditions are changed from high to low brightness. Since the average star is approximately white, the change of area of the luminosity-curve as its maximum shifts from $0.55\ \mu$ to $0.51\ \mu$ is slight, certainly much less than the uncertainty in choice of size of pupil.

It is of some interest to note that at the rate of energy-reception just calculated the eye receives through the pupil the elementary energy-quantum (6.585×10^{-27} erg. sec.$\times$frequency) in one-thousandth of a second.

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Philadelphia, July 1916

THE PHOTOGRAPHIC BRIGHTNESS OF THE FULL MOON

Father Hagen has very kindly called the writer's attention to a series of observations of the brightness of the moon at different phases which was overlooked in preparing the summary of the subject recently published.

These observations were made by Scheller[1] at Prag in 1906–1907, by comparison of the darkening of photographic plates by known exposures to direct moonlight and to a Hefner lamp at a distance of one meter—the law of photographic action of light of different intensities and for different exposures being determined for each plate by supplementary exposures. The observations were made and reduced with much care, and the results should be of high precision, although suffering somewhat from the difficulties which beset photometric work in the midst of a smoky city. Scheller calls attention to these difficulties, which caused him to reject the results obtained on two of the twenty-two nights of observation.

TABLE I

Phase	Nights	Difference of Magnitude between		O−C
		Moon and Lamp	Moon and Full	
−88°........	3	$+1^M.49$	$+2^M.36$	$+0^M.09$
−75°........	2	+1.19	+2.06	+ .23
−58°........	4	+0.53	+1.40	+ .08
−24°........	3	−0.35	+0.52	− .01
+ 5°........	3	−0.88	−0.01	− .13
+18°........	2	−0.17	+0.70	+ .26
+68°........	3	+0.64	+1.51	−0.29

The results for the remaining twenty nights, transformed into the notation used in the writer's paper, are summarized in Table I. To save space, the observations at similar phases are grouped into normals, giving each night equal weight. The first column gives the phase angle (negative before the full), the second the number of nights combined in forming the normal, and the third the mean

[1] *Sitzungsberichte der Akad. der Wissenschaften in Wien*, Math.-Wiss. Kl., 120, II, 889, 1911.

difference in stellar magnitude between the photographic brightness of the moon at the given phase, reduced to mean distance, and the Hefner lamp at one meter. On reducing these to mean full moon with the aid of the light-curve given in the writer's paper,[1] the photographic brightness of the full moon is found, from all the observations, to be $0^M.87 \pm 0^M.05$ greater than that of the lamp at one meter. The resulting differences in magnitude between the moon at the different phases and at the full are given in the fourth column, and the residuals from the light-curve previously mentioned in the last column.

These residuals appear to be due to accidental error of observation. Their average value, regardless of sign, but reduced to equal weight, is $\pm 0^M.13$ for the mean of three observations, corresponding to $\pm 0^M.23$ for a single observation. This is almost exactly equal to the corresponding quantity in the case of King's photographic observations, which was found to be $\pm 0^M.09$ for the mean of 6.4 observations, to which corresponds $\pm 0^M.23$ for one observation.[2]

If Scheller's observations were added to those employed in forming the normals of the previous paper (p. 116), with weight equal to King's, the residuals of the normals affected would be modified as follows:

Phase	−80°	−55°	−18°	+5°	+23°	+63°
Old residuals.......	$+0^M.04$	+0.04	+0.01	−0.04	−0.04	+0.01
New residuals......	+0.06	+0.05	+0.01	−0.06	−0.01	−0.03

The mean light-curve from which these residuals are taken would not therefore be sensibly altered by the inclusion of the additional observations.

It is, however, of much interest to note that, photographically, the full moon gives 2.2 times as much light as a Hefner lamp at a distance of a meter, while visually, according to Graff,[3] the light of the moon is only 0.27 that of the lamp, or $1^M.43$ fainter, as against $0^M.87$ brighter photographically. Hence the actinic power of moonlight, for equal visual intensities, is 8.3 times that of the

[1] *Astrophysical Journal*, **43**, 114, 1916.

[2] *Ibid.*, p. 116, 1916.

[3] *Ibid.*, p. 128.

lamplight. (Scheller, with a different reduction-curve, gets a ratio of 10.)

This may seem at first glance in glaring contradiction with the result derived by the writer, that the color-index of moonlight is $+1^{M}.18$, making its actinic intensity only one-third of the visual; but the apparent discrepancy is immediately explained when it is remembered that the standards of comparison in the two cases are very different, being in the first instance the yellowish-red flame, and in the second the stars of Class A, which are far whiter than any terrestrial sources of light.

From the foregoing data it appears that the color-index of the Hefner lamp is greater by $2^{M}.3$ than that of moonlight, and is therefore about $+3^{M}.5$ on the astronomical scale, so that, for equal visual intensities, the lamp has but $\frac{1}{25}$ the actinic power of the light of a star like Vega. This color-index is somewhat greater than that previously found[1] for the standard 2 c.-p. electric lamps studied by King which was $+2^{M}.9$; but it is of the same order of magnitude and appears entirely credible.

It may be added that Scheller's conclusion that the moon is brighter at the third quarter than at the first, which depends upon only three discordant observations near the former phase, is negatived by the concordant results of more than fifty observations by six different observers, near the same phase, which are discussed in the previous paper, and show beyond a doubt that the brightness is greater at the first quarter. The explanation which he suggests for his result—that the lunar maria, which cover a larger portion of the visible surface at the third quarter, reflect more actinic light than the rest of the disk—is disproved by a glance at any photograph of the moon.

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PRINCETON UNIVERSITY OBSERVATORY
June 16, 1916

[1] *Astrophysical Journal*, 43, 129.

Reviews

Stereoskopbilder von Sternhimmel. 2. Serie. Von MAX WOLF. Leipzig: J. A. Barth, 1915. M. 5.

The ordinary stereoscopic pictures—landscapes, etc.—are usually made by simultaneous exposures with two lenses several inches apart, thus obtaining the necessary parallactic effect which is given by the human eyes. If there are no moving objects in the view, the same result may be obtained with a single lens that is shifted a few inches between the two exposures. On account of the vast distances of the heavenly bodies the first of these methods is not possible when applied to the sky, except in the case of a meteor, which may be caught by two cameras some distance apart. The second method is applicable in a way for this purpose—the parallactic displacement being obtained either by the motion of the earth or of the object, or both, in the interval between the two photographs. To obtain the motion which will cause the required displacement, the time element in this method is a necessary factor. Beautiful results may thus be obtained in the case of a comet or a rapidly moving star, or the planets and satellites of the solar system. Though the result thus obtained seems in a manner to imitate the perspective given by the ordinary stereoscopic view, it is not strictly comparable with it in reality, for in the case of the rapidly moving star the appearance of relative distance is false. The star may really be farther away (though this is not probable) than its immediate neighbors, which appear as a distant background for it. The comet may have changed its form between the time of the two photographs, so that a false perspective of its relative parts may be shown. The perspective of the comet itself with respect to the stars gives no definite idea of the true distance and is always a false representation of the actual distance. The satellites of a planet are almost certain to be shown in a false perspective, and what appears to be a nearer satellite may really be a more distant one, the effect depending on the relative apparent motions of the satellites at the time, and not on their relative distances.

We must not, however, discourage the study of stereoscopic pictures of the celestial bodies; far from it, for they are not only beautiful but sometimes very instructive, and may lead us to correct conclusions not

otherwise obtainable. What we must do is to guard against the deceptions that will be produced by this method, for seeing is not always believing in such cases. Indeed, very serious errors may be promulgated by too much faith in what the stereoscope shows us in the sky.

The foregoing remarks will apply with more or less force to the subject we now have for review, Professor Wolf's second series of stereoscopic pictures of the heavens.

This consists of twelve stereoscopic views of certain celestial objects, and is issued in a neat cardboard case, for use in the ordinary stereoscope.

They are from photographs made by Dr. Max Wolf with reflecting and refracting telescopes at the observatory at Heidelberg, Germany, and are a continuation of a previous set of twelve such views. Following is the table of contents: Tafel 1, "Stern mit Eigenbewegung"; Tafel 2, "61 Cygni"; Tafel 3, "Mondkugel"; Tafel 4, "Mondlandschaft"; Tafel 5, "Patroclus"; Tafel 6, "Uranus"; Tafel 7, "Der Spiralnebel im Bären"; Tafel 8, "Der Spiralnebel in den Jagdhunden"; Tafel 9, "Der Nebel im Orion"; Tafel 10, "Komet Morehouse (1908, November 16)"; Tafel 11, "Komet Morehouse (1908, November 10)"; Tafel 12, "Blick in die Milchstrasse."

These pictures have been made by the second method which I have mentioned, but instead of a displacement of the camera of a few inches, its point of view has been changed by many thousands of miles, and generally with long intervals of time between the pictures. It is only necessary to describe a few of these photographs, though each one is of interest. The series consists of twelve stereoscopic views of comets, nebulae, proper-motion stars, planets and satellites, the moon, and the Milky Way. Apparently these are intended for the general public, as a popular explanation (in German) accompanies each plate.

The remarkable astronomical work of Dr. Max Wolf needs no word of comment here—and only praise of the highest order could be given it. It is pleasing to see that this effort to bring some of it to the attention of the public in a popular form has been so successful.

Plates 1 and 2 give views of proper-motion stars, one of these being the celebrated star 61 Cygni. These stars stand out strongly in perspective from the regular background, but in each case this is an effect wholly due to motion and has nothing whatever to do with the true distance, the effect of which would be inappreciable in these pictures. Plate 1 is very pleasing, the small star standing out beautifully between the observer and the other stars. The interval in this case is fourteen years.

Plate 3 shows the nearly full moon in plastic relief. The interval is about one month. This, like most stereoscopic views of the moon, does not show our satellite as a true sphere, but gives the impression of a distorted globe, some parts of which are rounder than others. The fault is due to the want of a perfect combination of phase and libration at the time of taking the pictures. A proper combination of these should give a perfectly spherical appearance to the moon.

Plate 4, which shows the lunar craters along part of the terminator, does not seem to be a success, as the relief is not satisfactory.

Plate 5 shows the small asteroid Patroclus. Allowance having been made for the motion of the planet, that object appears suspended in space as a small point, while the slight motion has caused the stars to trail somewhat. This picture is very successful and impressive and is well worth the making.

Plate 6 is a very interesting picture. It shows the planet Uranus and its two outer satellites in strong relief. The image of the planet is marred by the rays produced by the support of the flat in the reflecting telescope, but the picture is otherwise excellent. As these rays are artificial, their removal would have added much to the picture without injuring its scientific value.

The pictures of the spiral nebulae shown in Plates 7 and 8 are also very interesting; that of M 51 is especially pleasing. The coils of the spirals seem to stand out in relief from each other. This, however, cannot be due to either motion or parallax, or true perspective. One gets essentially the same effect with one picture and monocular vision.

Plate 9 is of the great nebula of Orion, and makes a beautiful and effective picture. It shows some perspective, both in the stars and in the nebula, but this does not seem to be real.

Plates 10 and 11 are views of Morehouse's comet. No. 10 is especially beautiful and the appearance of perspective is finely shown. It is the most effective picture of the set and is the more realistic from the fact that the exposures were so short that the motion of the comet during each exposure was not enough to elongate the star-images, so that they appear as points of light instead of trails. This photograph is to be highly commended.

Plate 12 is a fine view of the Milky Way in Aquila, made with a small lens. The perspective is good and the different distances of the various cloud-forms are well shown. One seems to be looking into the great depths of the Milky Way, although both pictures are made from

the same negative! It is an excellent example of how misleading stereoscopic views of the sky may sometimes be. With this fact in mind one must look again at the pictures of the nebulae shown in Plates 7, 8, and 9, and consider what the apparent perspective means. It shows how easily one can be misled by a false conclusion as to stellar displacements that do not exist except in the stereoscope itself. This is not intended as a criticism of the present work but as an explanation for those not familiar with the subject. As we have seen the case of the Milky Way photograph, two prints from the same negative may be used in the stereoscope with good effect. The writer has thus employed two prints from the negative of Saturn made with the 60-inch reflector at Mount Wilson, on 1911 November 19. Though there can be no possible true perspective in these pictures, they assume a pseudo perspective that greatly improves the view. When the two are thus seen in a stereoscope the result shows the planet as a ball suspended in the middle of the rings in excellent relief. It is a great improvement over the flatness that is so evident with the ordinary view.

If the limitations which have been pointed out are borne in mind, the photographs are to be commended, not only to the public, but also to astronomers.

While on the subject of stereoscopic vision it may not be out of place to call attention to some peculiarities of monocular vision that have come to the notice of the present writer. It is a subject with which very few seem to be familiar. Those who see the splendid perspective shown with the stereoscope are likely to express regret that one who has lost an eye is debarred from the use of these beautiful pictures. In a sense our sympathies are just. He who has lost an eye cannot use the stereoscope and hence loses some of their beautiful perspective. In reality, however, he is the only one who correctly sees a single photograph of a landscape or other subject—or a painting. The writer has been familiar with this fact for many years and has found much pleasure and instruction in it. The one who uses both eyes does not see such objects to the best advantage. He can put himself on an equal with the one-eyed man by closing one eye when he looks at most of the ordinary pictures.

Take any photograph, reproduction, or painting, where the perspective of distance is shown. View this at the ordinary distance; close one eye. The background will at once recede to its proper distance and the various objects will assume their regular perspective and you will have, in effect, a stereoscopic view. Open both eyes and the picture

at once becomes flat and the distances are in one plane—that of the photograph. Try this method on a picture where reflections are shown in a stream or pool of water. There will be a "sheen" on the water, which is such a beautiful feature in the stereoscope and which is so like what one sees in nature itself. Open both eyes and this "sheen" will disappear. I have often received additional pleasure by viewing with only one eye those exquisite photographs of mountain and other scenery so often given in the *National Geographic Magazine*. The difference between what is seen with one or two eyes is manifestly in favor of the one eye. Another point that strikes one in such examinations is that any particular object is more readily detected with one eye than with both. The one eye seems to pick out each individual object, so that when one is looking thus at a star photograph peculiarities not noticed with both eyes appear.

Space forbids my going further into the subject, but it is an interesting and important one. Of course we are doing here the same thing that we used to see done in art galleries, where the pictures were viewed with one eye, through a rolled-up pamphlet or tube of some kind. The tube is an added advantage because it excludes objects not intended as a part of the picture and which would otherwise detract from it.

E. E. BARNARD

YERKES OBSERVATORY, WILLIAMS BAY, WIS.
JULY 19, 1916

A Voyage in Space. By H. H. TURNER. London: Society for Promoting Christian Knowledge, 1915. Pp. 304, figs. 96. Cloth, 6*s*. net.

Professor Turner has presented us in book form with the course of "Juvenile Lectures" which he delivered at the Royal Institution during the Christmas time of 1913. The book is written in the author's usual attractive manner, and although the presentation of the different topics is clear enough in most instances for a child to grasp their meaning, folk of more mature years will find the book delightful reading. Many of the recent discoveries in astronomy are discussed and the chief landmarks in the progress of the science are reviewed. One of the main attractions of the book lies in the apt elucidations of difficult points; at this sort of thing Professor Turner is a master-hand. There are six lectures. The first lecture deals with the starting-point, our earth.

In the following lecture the length of the voyage and the start through the air are considered. The third lecture discusses the means of conveyance, the telescope. In the fourth lecture a "visit" to the moon and the planets is made, and in the fifth and sixth lectures the voyage is concluded by a visit to the sun and the stars. The volume could well be used as an introduction to astronomy for lay readers.

C. C. Crump